Linear Systems I
Final Exam

Notes: 1. Exam is closed-book and closed-notes.
2. You are allowed three 8.5x11 sheet of notes.
3. You are also allowed L.T., F.T. and Convolution table.
4. Show work for partial credit.
5. Please write clearly.

1. \( f(t) = u(t) \)

\[ p(t) = e^{-at} \cdot u(t), \quad a > 0 \]

\[ y(t) \]

Using convolution, find \( y(t) \). Also sketch \( y(t) \).

2. \( f(t) \)

Draw the exponential and trigonometric Fourier spectra of the periodic signal \( f(t) \) shown in Figure 2.

Figure 2

3. (a) Find the Fourier transform of

(b) Find the inverse Fourier transform of

\( f(t) \)

\( f(t) \)

\( F(w) \)

\( f(t) \)

\( F(w) \)
4. For the circuit shown in Figure 4, find $I(s)$.

\[ f(t) = e^{-t}u(t) \]

Note: Do not find the inverse Laplace transform of $I(s)$.

5. A mechanical system is described by the differential equation:

\[
\frac{d^2y}{dt^2} + 5 \frac{dy}{dt} + 6y = f(t) + \frac{df(t)}{dt}
\]

Find the unit impulse response of the system (i.e., $f(t) = \delta(t)$) described by the differential equation given above. Assume I.C.'s = 0

6 (a) i) Find the inverse Laplace transform of

\[
F(s) = \frac{e^{-s} - e^{-4s}}{s + 2}
\]

(3 pts) (ii) Plot $f(t)$ obtained in part (i)

(b) Using the initial value and final value theorems, find the initial and final values of the output $y(t)$. 