OBJECTIVE: The objective of this laboratory is to illustrate the definition of numerical integration as the area under curve.

EDUCATIONAL OBJECTIVES:
After performing this experiment, students should be able to:
1. Understand the basic definition of numerical integration as the area under the curve.
2. Be able to understand the concept of work done on a spring.
3. Use the symbolic toolbox and commands of integrations in Matlab.
4. Validate the mathematical model with observed data.

BACKGROUND:
In this lab, we will calculate the work done on the spring when a mass is attached to it. The force, $F$, exerted by the mass on the spring can be written using the Hooke's law as

$$F = k \cdot x$$

where $x$ is the displacement of the spring from an equilibrium position and $k$ is a constant characteristic of the particular spring known as spring constant. When a mass $m$ is attached to the spring as shown in the fig 1, it is stretched from its original length $x = l$ by a distance $\Delta x$. The work done on the spring is obtained by integrating the force and is given by.

$$\text{Work done} = \int_{0}^{\Delta x} F(x) dx$$

Figure 1: Spring and Mass System
PROCEDURE:

1. With the spring fixed at one end, attach a mass to the other end and add enough mass to expand the spring by a small distance from its completely closed state. Measure the displacement of the spring form the initial point accurately. Record your measurement.

2. Now, add a total of 160 gm mass in 2 increments of 80 gm each. Let the oscillations die down. Measure the displacement of the spring after each increment from the initial point accurately.

3. Remove all the masses and add the same 160 gm of mass incrementally in steps of 40 gm each to expand the spring. This can be done as follows. Add 40 gm of mass to the spring and measure the displacement (X₁) after the oscillations die down. Then, add 40 gm of mass to the existing 40 gm and measure the new displacement (X₂). Continue the process till the total mass attached to the spring is 160 gm. Record the displacement after each increment in Table I.

4. Finally, add 160 gm of mass (similar to step 3) incrementally in the steps of 20 gm each to expand the spring. Each time when the mass of 20 gm is added, tabulate the displacements. Make sure the oscillations have died down before taking the displacement reading.

5. Calculate the force (in N.m) applied to the spring by converting the mass into weight in Table I.

6. Plot the total displacement (x-axis) versus the total force (y-axis) after each incremental mass and find the spring constant, k, using part III of the Table I.

7. Now, plot a bar graph of displacement vs. force for all three cases.

8. For each case, add the areas of the bars from the bar graph to get the total work done on the spring.

9. Now, calculate the total work done on the spring by evaluating the integral (by hand) in equation (2)

\[
W = \int_0^{\Delta x} F(x)dx = \int_0^{\Delta x} (k \cdot x)dx
\]  

Use the value of k determined in part 6 of the procedure and \(\Delta x\) as the final measured displacement.

<table>
<thead>
<tr>
<th>Part I</th>
<th>Part II</th>
<th>Part III</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\Delta m)</td>
<td>X</td>
<td>M</td>
</tr>
<tr>
<td>0</td>
<td>R=</td>
<td>0</td>
</tr>
<tr>
<td>80</td>
<td>X₁=</td>
<td>80</td>
</tr>
<tr>
<td>80</td>
<td>X₂=</td>
<td>160</td>
</tr>
<tr>
<td>40</td>
<td>X₃=</td>
<td>120</td>
</tr>
<tr>
<td>40</td>
<td>X₄=</td>
<td>160</td>
</tr>
<tr>
<td>20</td>
<td>X₅=</td>
<td>100</td>
</tr>
<tr>
<td>20</td>
<td>X₆=</td>
<td>120</td>
</tr>
<tr>
<td>20</td>
<td>X₇=</td>
<td>140</td>
</tr>
<tr>
<td>20</td>
<td>X₈=</td>
<td>160</td>
</tr>
</tbody>
</table>

*Table I.* Displacement and force calculations after each incremental mass is attached to the spring.
Calculation of work done using MATLAB:

Evaluate the integral in equation (2) and draw the bar graph of total work done (part 8 of the procedure) using the following Matlab commands.

Matlab Commands:

To define x as a variable:

```
syms x
```

To integrate k*x from 0 to Δx:

```
int (k*x,0, Δx)
```

To draw a bar graph with bars touching one another:

```
bar(x,y,1)
```

LAB QUESTION

*Compare the calculated work done from the three cases of the experiment with the work done obtained from Matlab. Explain why the result obtained from Part III is closer to the Matlab result as compared to the results of Part I and II.*