PS1. The profile of a gear tooth is approximated as a half sinusoid:

\[ y(x) = h \sin \frac{\pi x}{l} \]

Area, \( A \)

a) Estimate the area \( A \) using 6 inscribed rectangles of equal width \( \Delta x = \frac{l}{6} \):

b) Calculate the exact area by evaluating the definite integral:

\[ A = \frac{l}{2} \int_{-l/2}^{l/2} y(x) \, dx \]
A particle is accelerated along a curved path of length \( l \) under the action of an applied force \( f(x) \):

\[
\int f(x) \, dx
\]

The total work done on the particle is

\[
W = \int_0^l f(x) \, dx \quad \text{N.m}
\]

e.g. \( l = 2.0 \, \text{m} \), determine the work done for:

a) \( f(x) = 4x^3 + 3x^2 + 2x + 1 \, \text{N} \)

b) \( f(x) = 2e^{2x} \, \text{N} \)

c) \( f(x) = 2 \sin(2x) \cos(2x) \, \text{N} \)

Hint on part c): I think they mean identity.
53. An area is bounded by a straight line in the x-y plane.

![Diagram of area A]

a) Find the equation of the line $y(x)$.

b) Find the area $A$ by integration,

$$A = \int y(x) \, dx$$

c) Find the centroid $G$ by integration with respect to $x$:

$$\bar{x} = \frac{\int x \, y(x) \, dx}{A}$$

$$\bar{y} = \frac{\int y(x) \, dx}{A}$$

$$\bar{y} = \frac{\int \left(\frac{y(x)}{2}\right) \, dx}{A}$$

d) Now solve for $x$ as a function of $y$, and recalculate $\bar{y}$ by integration with respect to $y$:

$$\bar{y} = \int \frac{y \, dx}{A} = \int \frac{y \, x(y) \, dy}{A}$$
5.4. An area in the $x$-$y$ plane is bounded by the curve $y = \frac{1}{2}x^2$ and the line $x = 6$.

![Graph showing curve $y = \frac{1}{2}x^2$ and line $x = 6$.]

a) Determine the area $A$ by integration.

b) Determine the coordinates of the centroid $G$ by integration.

5.5. A beam is subjected to a quadratic distributed load:

![Diagram of a beam with quadratic load $w(x)=w_0 x(x-l)$.

a) Determine the total resultant force $R = \int_0^l w(x) \, dx$.

b) Show that the resultant $R$ acts at the point $x = l/2$ (i.e., show that the centroid of the distributed load is $x = l/2$).