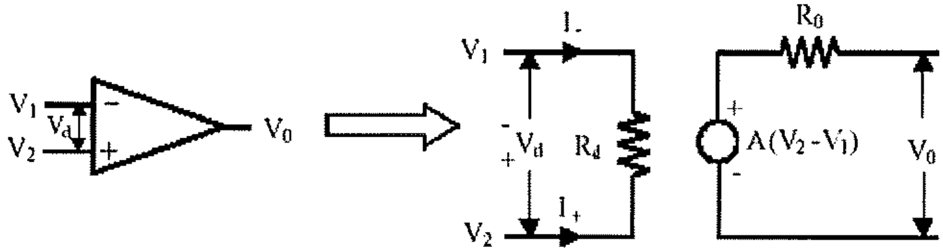
Operational Amplifiers (Op-Amps)



For ideal op-amps, $A = \infty$, $R_d = \infty$, $R_0 = 0$, and $V_-(V_1) = V_+(V_2)$, $I_- = I_+ = 0$.

For real op-amps, $A \neq \infty$, $R_d \neq \infty$, $R_0 \neq 0$, and $V_-(V_1) \neq V_+(V_2)$, $I_- \neq 0$, $I_+ \neq 0$.

Basic Simplifying Rules

$V_+ = V_-$

$I_+ = I_- = 0$ (no current flow into either input terminal)

Ideal Op-Amp Properties

Gain $A = \infty$

Input Impedance $R_d = \infty$

Output Impedance $R_o = 0$

Output Voltage $V_{out} = 0$, when $V_1 = V_2$ (no offset voltage)

Bandwidth $= \infty$ (no phase shift)

Basic Op-Amp Configurations

For Non-Inverting Configuration	$V_{out} = (1 + R_f / R_i) V_{in}$
For Inverting Configuration	$V_{out} = (-R_f / R_i) V_{in}$
Differential Configuration (Two Inputs - One Output)	$V_{out} = (-R_f / R_i)(V_1 - V_2)$

Common Mode Rejection

Common Mode Voltage	$V_{CMV} = (V_1 + V_2)/2$
Differential Voltage	$V_{DV} = V_1 - V_2$
Common Mode Gain	$CMG = \lvert V_{out} / V_{CMV} \rvert$
Common Mode Rejection Ratio	$CMRR = DG / CMG = 20 \log (DG/CMG)$ dB

Properties of Real Op-Amps

Open Loop Gain $A = A_0 / (1 + j\omega\tau)$ $\tau = 1/\omega_0 = 1/2\pi f_0$

for $f << f_0$ $A = A_0$
for $f >> f_0$ $A = A_0 (f_0 / f)$

Typical Values:

$A_0 = 10^5$
$f_0 = 10$ Hz

At some frequency f, $|A| = 1$, and $B = f$ is the so-called *unit-gain frequency*

For 741 Op-Amp: $A_0 = 1\text{x}10^5$ $f_0 = 5$ Hz $B = 1$ MHz
For 411 Op-Amp: $A_0 = 2\text{x}10^5$ $f_0 = 10$ Hz $B = 4$ MHz

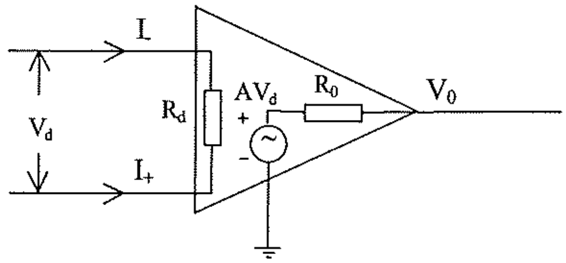
Closed Loop Gain (Negative Feedback)

For both Inverting and Inverting
Open Loop Gain $= A$
Feedback Loop Gain $= \beta = R_i / (R_i + R_f)$
Loop Gain $= A\beta$
Closed Loop Gain $= K$
Unit-Gain Frequency $= B$ $\{B = f$ when $|A| = 1\}$
Bandwidth $f_0 = B\beta$
Gain-Bandwidth Product $= A_0 f_0 = A_l f_l = B$

For Non-Inverting
$K = V_o / V_i = A_l = A / (1 + A\beta) \cong 1/\beta = 1 + R_f / R_i$

For Inverting
$K = V_o / V_i = A_l = A(\beta - 1)/(1 + A\beta) \cong (\beta - 1)/\beta = -R_f / R_i$
$K = V_o / V_i = A_l = A(1 - \beta)/(1 - A\beta) \cong (1 - \beta)/(-\beta) = 1 - 1/\beta = -R_f / R_i$

SUMMARY OF OP-AMP



I. Ideal Op-Amp

$A = \infty$, $R_d = \infty$, $R_0 = 0$

The above three approximations lead to the following two rules for circuit analysis:

$$V_- = V_+ \quad \text{and} \quad I_- = I_+ = 0$$

II. Real Op-Amp

1. A

$$A = \frac{A_0}{1 + j\frac{f}{f_0}}$$ where A is the open-loop gain ($A = V_0/V_d$), and f_0 is the corner frequency

For µA741, $A_0 = 2\text{x}10^5$, and $f_0 = 5$Hz. For µA411, $A_0 = 10^5$, and $f_0 = 40$Hz.

When an op-amp is used to build a feedback circuit, the following equation exists:

$$A_0 f_0 = A_1 f_1 \; (= A_2 f_2 = A_3 f_3 = \ldots..)$$

where $A_1 \cong 1/\beta$ is the close-loop gain (amplifier gain) and f_1 is the new corner frequency

2. R_d (typical value: 500 K ~ 2 M)

Input impedance R_{in} of a circuit:

Inverter: $R_{in} = R_i$
Non-inverter: $R_{in} = (1+A\beta)\, R_d$

3. R_0 (typical value: 100Ω)

Output impedance for both inverter and non-inverter: $R_{out} = R_0/(1+A\beta)$

4. Bias current I_b (t.v.: 80 nA), **differential bias current I_{i0}** (t.v.: 20 nA)

5. Offset voltage V_{i0} (typical value: 1 mv)

General Characteristics of Op-Amp Circuits

i_+ = Current into Non-Inverting Input
i_- = Current into Inverting Input

$$i_+ = i_- = 0$$

v_+ = Voltage at Non-Inverting Input
v_- = Voltage at Inverting Input

$$V_d = (v_+ - v_-)$$

$$V_{out} = A*V_d = A*(v_+ - v_-)$$

If $V_d = (v_+ - v_-) > 0$ that is to say: if $v_+ > v_-$ then V_{out} is positive.
Which is exactly the same as: if $v_- < v_+$ then V_{out} is positive.

If $V_d = (v_+ - v_-) < 0$ that is to say: if $v_+ < v_-$ then V_{out} is negative.
Which is exactly the same as: if $v_- > v_+$ then V_{out} is negative.

Examples:

$V_+ = +1.5$ V $V_- = +0.5$ V V_{out} Positive	$V_+ = +1.5$ V $V_- = -1.5$ V V_{out} Positive	$V_+ = -2.0$ V $V_- = -2.5$ V V_{out} Positive
$V_+ = +0.5$ V $V_- = +1.5$ V V_{out} Negative	$V_+ = -0.5$ V $V_- = +1.5$ V V_{out} Negative	$V_+ = -2.5$ V $V_- = -2.0$ V V_{out} Negative

And since A is very very large
Positive $V_{out} = +V_{sat} \approx (+Vcc - 1)$
Negative $V_{out} = -V_{sat} \approx (-Vcc + 1)$

Op-Amp as Simple Comparator (v_- Grounded)

$V_{in} = v_+$ and $v_- = GND = 0$.
If $V_{in} > 0$, then $V_{out} = +V_{sat}$
If $V_{in} < 0$, then $V_{out} = -V_{sat}$

Op-Amp as Simple Comparator (v_- Biased to $\pm V_{ref}$ from voltage divider and battery supply)

$V_{in} = v_+$ and $v_- = +V_{ref}$
If $V_{in} > V_{ref}$, then $V_{out} = +V_{sat}$
If $V_{in} < V_{ref}$, t hen $V_{out} = -V_{sat}$

$V_{in} = v_+$ and $v_- = -V_{ref}$
If $V_{in} > -V_{ref}$, then $V_{out} = +V_{sat}$
If $V_{in} < -V_{ref}$, then $V_{out} = -V_{sat}$

Op-Amp as Schmitt Trigger (v_+ Biased to V_{ref} from voltage divider and V_{out})

Define V_{TH} (Threshold Voltage High) = $+V_{ref}$ (when V_{out} is positive)
and V_{TL} (Threshold Voltage Low) = $-V_{ref}$ (when V_{out} is negative)
$v_- = V_{in}$ and $v_+ = V_{ref} = +V_{TH}$ for V_{out} Positive or $v_+ = V_{ref} = -V_{TL}$ for V_{out} negative

If V_{out} positive and $V_{in} < V_{TH}$ (i.e., $v_- < v_+$) then V_{out} stays positive ($V_{out} = +V_{sat}$).
If V_{out} positive and $V_{in} > V_{TH}$ (i.e., $v_- > v_+$) then V_{out} switches to negative ($V_{out} = -V_{sat}$).
If V_{out} negative and $V_{in} > V_{TL}$ (i.e., $v_- > v_+$) then V_{out} stays negative ($V_{out} = -V_{sat}$).
If V_{out} negative and $V_{in} < V_{TL}$ (i.e., $v_- < v_+$) then V_{out} switches to positive ($V_{out} = +V_{sat}$).
Switches from positive to negative when $V_{in} > V_{TH}$ (i.e., $v_- > v_+$).
Switches from negative to positive when and $V_{in} < V_{TL}$ (i.e., $v_- < v_+$).

Op-Amp Feedback Analysis Rules

Inverting amplifiers are configured such that the input is applied to the inverting terminal.

Non-inverting amplifiers are configured such that the input is applied to the non-inverting terminal.

Open-Loop pertains to configurations without feedback loops.
For the case of open loop (no feedback) Vout = A(V+ - V-) where Vout Max = Vsupply.

Closed-Loop pertains to configurations with a feedback loop.
Feedback can be supplied to either the inverting input (negative feedback) or to the non-inverting input (positive feedback). In general, only negative feedback (inverting input) is considered. Positive feedback is usually associated with oscillator circuits or special comparator circuits such as Schmitt Triggers.
In comparator circuits, the output is driven towards the supply voltage limits (saturation voltage).
The current rule (Rule One, see below), applies to almost all op-amp configurations.
The voltage rule (Rule Two, see below) only applies to negative feedback configurations.

For closed-loop (negative feedback) with Vin applied to the inverting terminal: Vout/Vin = -Rf/Rs.

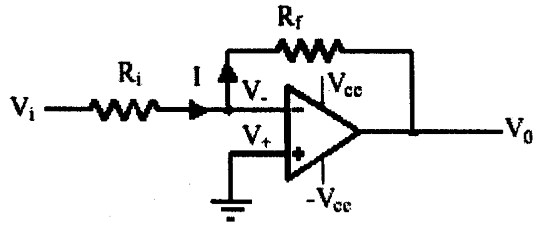
For closed-loop (negative feedback) with Vin applied to the non-inverting terminal: Vout/Vin = 1 + Rf/Rs.

Rule One: i- = i+ = 0
This is true for ideal op-amps and is essentially the case for real op-amps as well.
The input resistance for real op-amps is on the order of tera-ohms (that's 10^{12}),
so the input current is in the range of pico-amps (that's 10^{-12} or 0.000000001 milli-amps)
which can be considered to be zero for our calculations!

Rule Two: V+ = V-
This rule only applies when the op-amp is configured with a negative feedback loop.
For negative feedback, whenever the op-amp senses a voltage difference between its inverting and non-inverting inputs, it responds by feeding back as much current/voltage through the feedback network as is necessary to keep the difference between V+ and V- equal to zero; that is to say,
V+ - V- = 0 and therefore V+ = V-.

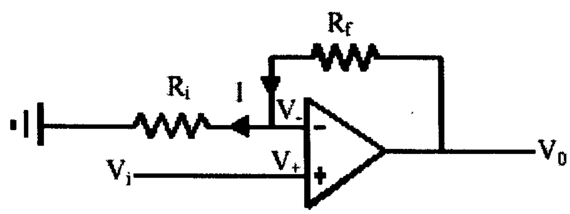
Note:
Rule Two does not say that V+ = V- = 0; but only states V+ = V-.
If either V+ or V- is connected directly to ground, then V+ = V- = 0 !!!

Inverting Op-Amp



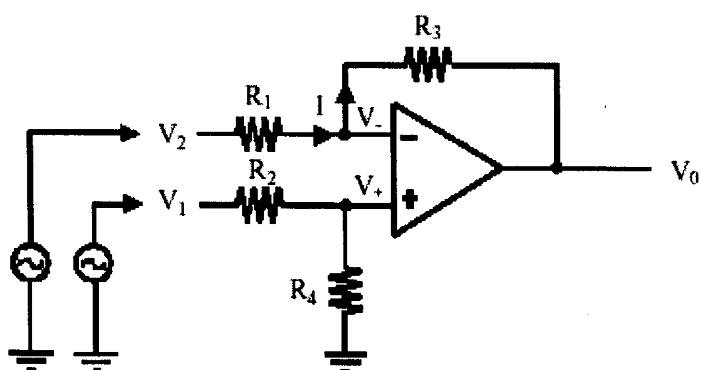
$$V_0 = -\frac{R_f}{R_i} V_i$$

Non-Inverting Op-Amp



$$V_0 = \left(1 + \frac{R_f}{R_i}\right) V_i$$

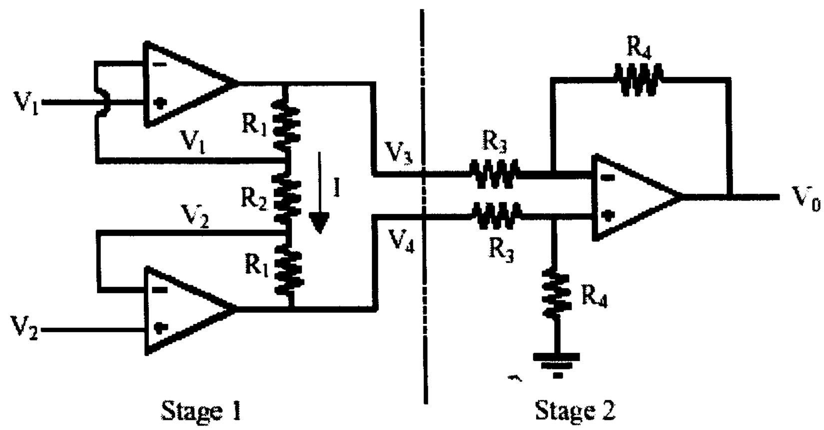
Differential Amplfier



$$V_0 = \frac{R_f}{R_i}(V_2 - V_1)$$

Instrumentation Amplifier (Three Op-Amp - Two Stage)

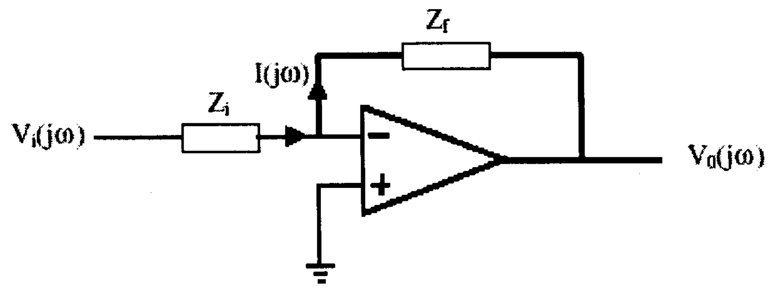
DG = $(1 + 2R_1 / R_2)(R_4 / R_3)$
CMG = 0
Very High Impute Impedance (draws very little current from source)
Very High CMRR
Differential Gain (easily adjustable via one potentiometer {R_2 in figure below})



Instrumentation Amplifier

$$\frac{V_0}{V_2 - V_1} = \left(1 + 2\frac{R_1}{R_2}\right)\frac{R_4}{R_3}$$

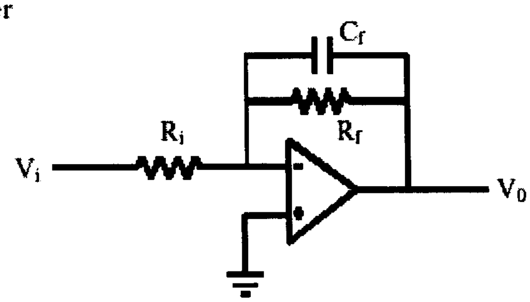
Active Filters (with Gain)



The transfer function of the circuit is:

$$K(j\omega) = \frac{V_0(j\omega)}{V_i(j\omega)} = -\frac{Z_f}{Z_i}$$

Low-pass filter

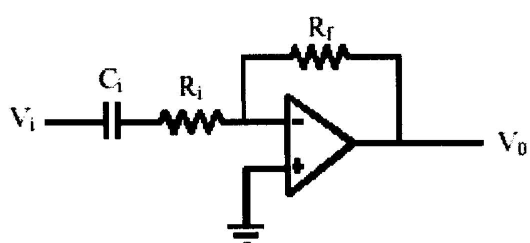


Using the frequency-domain analysis, $Z_i = R_i$, $Z_f = R_f // \frac{1}{j\omega C_f} = \frac{R_f}{1 + j\omega R_f C_f}$

$$K(j\omega) = -\frac{R_f}{R_i}\frac{1}{1 + j\omega R_f C_f} = \frac{a}{1 + j\omega\tau}$$

where $a = -R_f / R_i$ and $\tau = R_f C_f$

High-pass filter



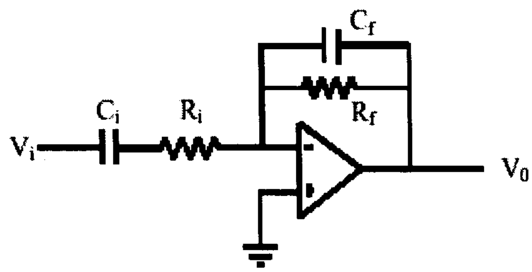
Again, using the frequency-domain analysis, $Z_i = R_i + 1/j\omega C_i$ and $Z_f = R_f$

Therefore, $$K(j\omega) = -\frac{Z_f}{Z_i} = -\frac{R_f}{R_i + \frac{1}{j\omega C_i}} = -\frac{j\omega R_f C_i}{1 + j\omega R_i C_i}$$

$$= -\frac{R_f}{R_i}\frac{j\omega\tau}{1 + j\omega\tau} = a\frac{j\omega\tau}{1 + j\omega\tau}$$

where $a = -R_f/R_i$ and $\tau = R_i C_i$

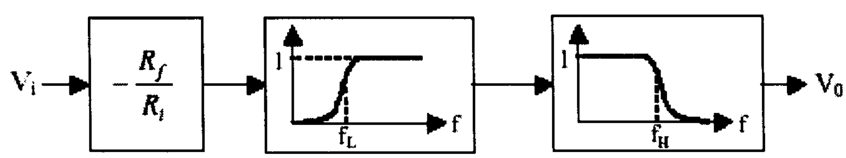
Band-pass filter



$$K(j\omega) = -\frac{Z_f}{Z_i} = -\frac{R_f \,//\, \frac{1}{j\omega C_f}}{R_i + \frac{1}{j\omega C_i}} = -\frac{R_f}{R_i} \cdot \frac{j\omega\tau_i}{1 + j\omega\tau_i} \cdot \frac{1}{1 + j\omega\tau_f}$$

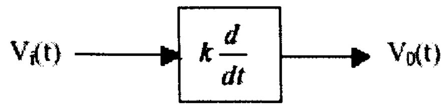
where $\tau_i = R_i C_i$ and $\tau_f = R_f C_f$. We must have $\tau_f < \tau_I$

From Eq. (5-26), a band-pass filter can be considered as a cascade of three blocks: a gain of $-R_f/R_i$, a high-pass filter, and a low-pass filter, as shown below.



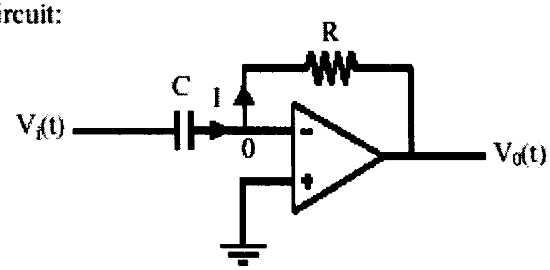
Gain HP: $f_L = \frac{1}{2\pi\tau_i}$ LP: $f_H = \frac{1}{2\pi\tau_f}$

Differentiator



For a differentiator, the input-output relation is: $V_0(t) = K \dfrac{dV_i(t)}{dt}$

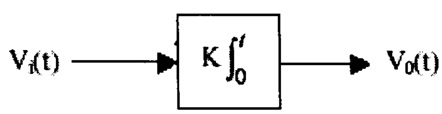
Actual circuit:



We have: $I = C\dfrac{dV_c}{dt} = C\dfrac{dV_i(t)}{dt}$, and $0 - V_0(t) = I\,R$

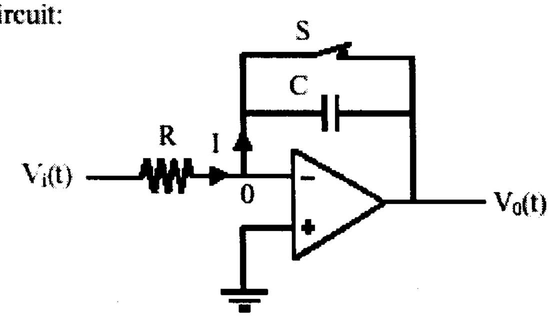
Therefore, $V_0(t) = -\,RC\dfrac{dV_i(t)}{dt}$ and the circuit is a differentiator.

Integrator



For a differentiator, the input-output relation is: $V_0(t) = K\int_0^t V_i(\hat{o})d\hat{o}$

Actual circuit:

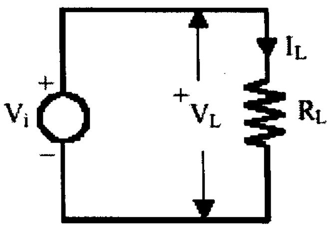


For $t < 0$, the switch S is closed. $V_0(t) = 0$. At $t = 0$, S opens.

We then have: $V_0(t) = -\dfrac{1}{C}\int_0^t I(\hat{o})d\tau = -\dfrac{1}{RC}\int_0^t V_i(\tau)d\tau$

Voltage-Controlled Current Source (VCIS)

Comparisons between a voltage source and a current source:



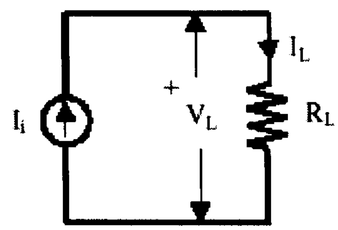
Voltage source

i) $V_L = V_i$ is constant, independent of R_L

ii) $I_L = \frac{V_i}{R_L}$ depends on R_L

$I_L \le I_{Limit}$

iii) Open circuit is O. K.
Can't have short circuit



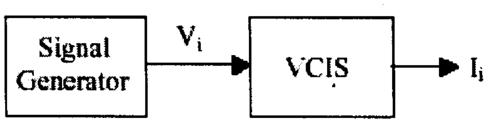
Current source

i) $I_L = I_i$ is constant, independent of R_L

ii) $V_L = I_i \cdot R_L$ depends on R_L

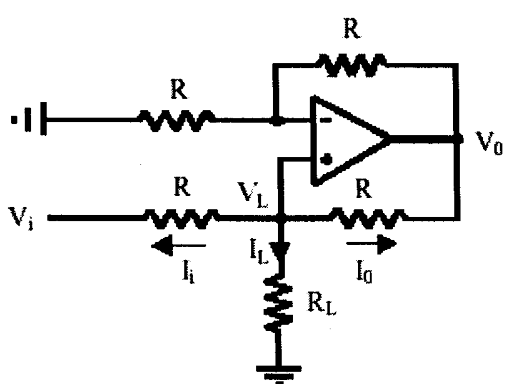
$V_L \le V_{Limit}$

iii) Short circuit is O.K.
Can't have open circuit



$g_m = \frac{I_i}{V_i}$ Transductance (having units of Ω^{-1})

VCIS with grounded terminals



Active Filters Using Op-Amps

Low Pass Filter (Inverting Input)

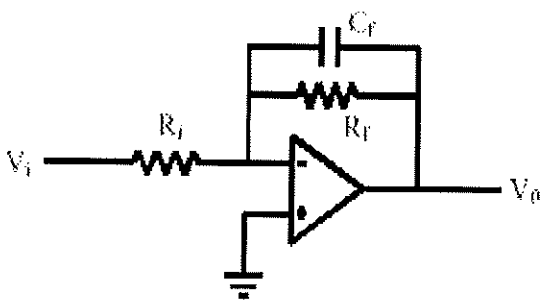
$\alpha = -R_f / R_i$

$\tau = R_f C_f$

Cut-Off Freq $= 1 / 2\pi R_f C_f$

Pass Band Gain $= 20\log_{10}(-R_f / R_i)$ dB

$$K(j\omega) = \frac{\alpha}{1 + j\omega\tau}$$



Low Pass Filter (Non-Inverting Input)

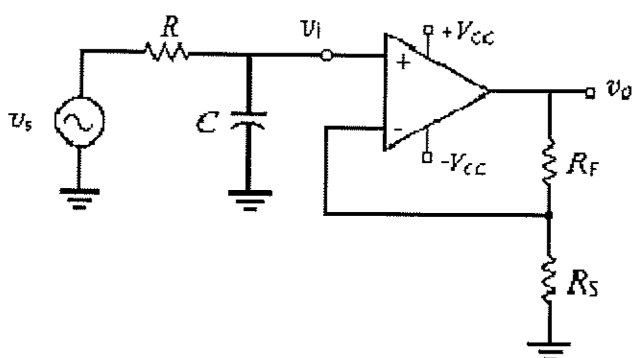
$\alpha = 1 + R_f / R_s$

$\tau = RC$

Cut-Off Freq $= 1 / 2\pi RC$

Pass Band Gain $= 20\log_{10}(1 + R_f / R_i)$ dB

$$K(j\omega) = \frac{\alpha}{1 + j\omega\tau}$$



Active Filters Using Op-Amps

High Pass Filter (Inverting Input)

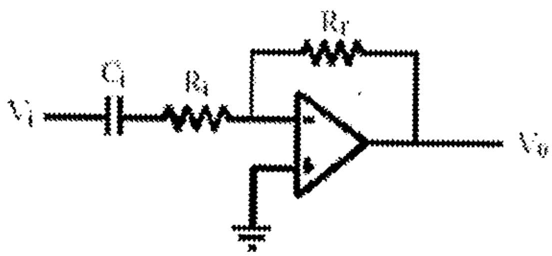
$\alpha = -R_f / R_i$

$\tau = R_i C_i$

Cut-Off Freq = $1 / 2\pi R_i C_i$

Pass Band Gain = $20\log_{10}(-R_f / R_i)$ dB

$$K(j\omega) = \frac{\alpha}{1 + \frac{1}{j\omega\tau}}$$



High Pass Filter (Non-Inverting Input)

$\alpha = 1 + R_f / R_s$

$\tau = RC$

Cut-Off Freq = $1 / 2\pi RC$

Pass Band Gain = $20\log_{10} (1 + R_f / R_i)$ dB

$$K(j\omega) = \frac{\alpha}{1 + \frac{1}{j\omega\tau}}$$