RESISTOR, INDUCTOR, CAPACITOR

When electrical energy is supplied to a circuit element, it will respond in one or more of the following three ways. If the energy is consumed, then the circuit element is a pure *resistor*. If the energy is stored in a magnetic field, the element is a pure *inductor*. And if the energy is stored in an electric field, the element is a pure *capacitor*. A practical circuit device exhibits more than one of the above and perhaps all three at the same time, but one may be predominant. A coil may be designed to have a high inductance, but the wire with which it is wound has some resistance; hence the coil has both properties.

RESISTANCE R

The potential difference $v(t)$ across the terminals of a pure resistor is directly proportional to the current $i(t)$ in it. The constant of proportionality R is called the resistance of the resistor and is expressed in volts/ampere or ohms.

$$v(t) = R\,i(t) \qquad \text{and} \qquad i(t) = \frac{v(t)}{R}$$

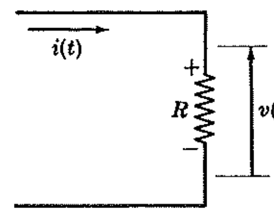


Fig. 1-3

No restriction is placed on $v(t)$ and $i(t)$; they may be constant with respect to time, as in D.C. circuits, or they may be sine or cosine functions, etc.

Lower case letters (v, i, p) indicate general functions of time. Capital letters (V, I, P) denote constant quantities, and peak or maximum values carry a subscript (V_m, I_m, P_m).

INDUCTANCE L

When the current in a circuit is changing, the magnetic flux linking the same circuit changes. This change in flux causes an emf v to be induced in the circuit. The induced emf v is proportional to the time rate of change of current if the permeability is constant. The constant of proportionality is called the *self-inductance* or *inductance* of the circuit.

$$v(t) = L\frac{di}{dt} \qquad \text{and} \qquad i(t) = \frac{1}{L}\int v\,dt$$

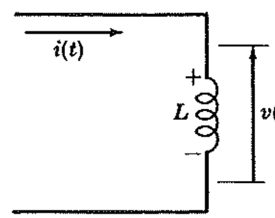


Fig. 1-4

When v is in volts and di/dt in amperes/sec, L is in volt-sec/ampere or *henries*. The self-inductance of a circuit is 1 henry (1 h) if an emf of 1 volt is induced in it when the current changes at the rate of 1 ampere/sec.

CAPACITANCE C

The potential difference v between the terminals of a capacitor is proportional to the charge q on it. The constant of proportionality C is called the *capacitance* of the capacitor.

$$q(t) = C\,v(t), \quad i = \frac{dq}{dt} = C\frac{dv}{dt}, \quad v(t) = \frac{1}{C}\int i\,dt$$

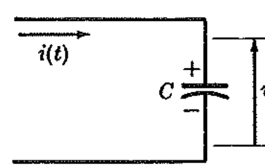


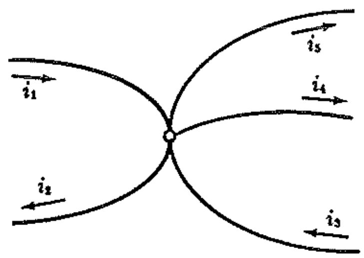
Fig. 1-5

When q is in coulombs and v in volts, C is in coulombs/volt or *farads*. A capacitor has capacitance 1 farad (1 f) if it requires 1 coulomb of charge per volt of potential difference between its conductors. Convenient submultiples of the farad are

$$1\,\mu\text{f} = 1 \text{ microfarad} = 10^{-6}\text{ f} \quad \text{and} \quad 1\,\mu\mu\text{f} = 1 \text{ micromicrofarad} = 10^{-12}\text{ f}$$

KIRCHHOFF'S LAWS

1. The sum of the currents entering a junction is equal to the sum of the currents leaving the junction. If the currents toward a junction are considered positive and those away from the same junction negative, then this law states that the algebraic sum of all the currents meeting at a common junction is zero.

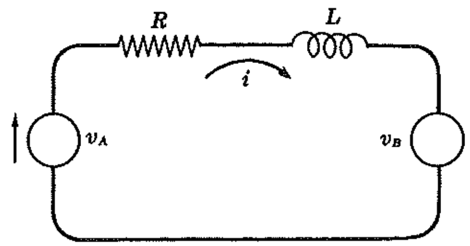


Σ currents entering $=$ Σ currents leaving

$$i_1 + i_3 = i_2 + i_4 + i_5$$

or $$i_1 + i_3 - i_2 - i_4 - i_5 = 0$$

Fig. 1-6



Σ potential rises $=$ Σ potential drops

$$v_A - v_B = Ri + L(di/dt)$$

or $$v_A - v_B - Ri - L(di/dt) = 0$$

Fig. 1-7

2. The sum of the rises of potential around any closed circuit equals the sum of the drops of potential in that circuit. In other words, the algebraic sum of the potential differences around a closed circuit is zero. With more than one source when the directions do not agree, the voltage of the source is taken as positive if it is in the direction of the assumed current.

Circuit Response of Single Elements

Element	Voltage across element	Current in element
Resistance R	$v(t) = R\,i(t)$	$i(t) = \frac{v(t)}{R}$
Inductance L	$v(t) = L\frac{di}{dt}$	$i(t) = \frac{1}{L}\int v\,dt$
Capacitance C	$v(t) = \frac{1}{C}\int i\,dt$	$i(t) = C\frac{dv}{dt}$

Units in the MKS System

Quantity		Unit		Quantity		Unit	
Length	l	meter	m	Charge	Q, q	coulomb	c
Mass	m	kilogram	kg	Potential	V, v	volt	v
Time	t	second	sec	Current	I, i	ampere	amp
Force	F, f	newton	nt	Resistance	R	ohm	Ω
Energy	W, w	joule	j	Inductance	L	henry	h
Power	P, p	watt	w	Capacitance	C	farad	f

Electrical Theory

Quantity	Symbol	Unit	Equation
Charge	Q	coulomb	$Q = \int I dt$ $Q = CV$
Current	I	ampere	$I = dQ/dt$
Voltage	V	volt	$V = dW/dQ$
Energy	W	joule	$W = \int V dQ = \int P dt$
Power	P	watt	$P = dW/dt = IV$

1

Electrical Theory - continued

Quantity	Symbol	Unit	Equation
Resistor	R	ohm	$V = IR$
Inductor	L	henry	$V = L\, dI/dt$ $I = 1/L \int V dt$
Capacitor	C	farad	$V = 1/C \int I dt$ $I = C\, dV/dt$

2

Electrical Theory

Ohm's Law
- $I = V/R$ (DC) $I = V/Z$ (AC)

Kirchoff's Law
- Sum of Loop Voltages = 0
- Sum of Node Currents = 0

Joule's Law
- $P = IE = I^2R$ (I Squared R Loss)

3

Current Flow

Electron Flow - From excess to deficient

Conventional Current Flow
- Internal to Source (Battery)
  - Negative to Positive
- External from Source (Battery)
  - Positive to Negative

Voltage Drop - Across a Resistor + to -

Negative Current - Assumed Direction Reversed

4

Equivalent Circuits

Thevenin
- Two Terminal Resistor and Battery Circuit
  - Series Voltage Source and Equivalent Resistor
- Voltage Source = Open Circuit Voltage
- Equivalent Resistor = V / Short Circuit Current

Norton
- Two Terminal Resistor and Battery Circuit
  - Parallel Voltage Source and Equivalent Resistor
- Current Source = Short Circuit Current
- Equivalent Resistor = V / Short Circuit Current

Series and Parallel Components

Component	Series	Parallel
R	$R_{eq} = R_1 + R_2 + R_3$	$1/R_{eq} = 1/R_1 + 1/R_2 + 1/R_3$
Z	$Z_{eq} = Z_1 + Z_2 + Z_3$	$1/R_{eq} = 1/Z_1 + 1/Z_2 + 1/Z_3$
L	$L_{eq} = L_1 + L_2 + L_3$	$1/L_{eq} = 1/L_1 + 1/L_2 + 1/L_3$
C	$1/C_{eq} = 1/C_1 + 1/C_2 + 1/C_3$	$C_{eq} = C_1 + C_3 + C_3$

6

DC Circuit Components

Component	Impedance	Current	Power/Energy
R	R	$I = V/R$	$I^2 R$
L	Zero	Infinite	$1/2\ LI^2$
C	Infinite	Zero	$1/2\ CV^2$

7

AC Sinusoidal Analysis

Resistor	R	$I = V/R$	$P = I^2R = V^2/R$
Inductor	$X_L = j\omega L$	$I = -jV_L/\omega L$	$Q_L = I^2X_L = V_L^2/X_L$
Capacitor	$X_C = -j/\omega C$	$I = jV_C\omega C$	$Q_C = I^2X_C = V_C^2/X_C$

Current $I = I_R + j I_X$

Voltage $V = V_R + j V_X$

Complex Power $S = VI^* = (V_R + j V_X)(I_R - j I_X)$

Complex Power $S = P + j Q$

8

Power and Voltage Ratios Expressed in Decibels (dB's)

1 Bel = log(Power2 / Power1)

1 decibel = 1 dB = 0.1 Bel, hence 10 dB = 1 Bel

To express a Power Ratio in dB's, use dB = 10 log(Power2 / Power1)

Let Power2 =2 Power1
Power Ratio in dB's = 10 log(2 Power1 / Power1) = 10 log(2) = +3.01

Let Power2 = 0.5 Power1
Power Ratio in dB's = 10 log(0.5 Power1 / Power1) = 10 log(0.5) = -3.01
-3 dB is often expressed as "3 dB Down" which is the half power point (Power2 = 1/2 Power1)

Let Power2 = Power1
Power Ratio in dB's = 10 log(Power1 / Power1) = 10 log(1) 0
dB = 0 does not imply zero power but rather a power ratio of one-to-one
dB = 0 can be used as a zero reference; that is to say, set your reference level to a particular value and then use the dB scale to refer all other values to that reference level.

Examples: Reference Level = 400 watts.
200 watts = -3 dB
800 watts = +3 dB
400 watts = 0 dB
4000 watts = +l0 dB
40 watts = -l0 dB
650 watts = +2.1 dB
65 watts = -7.9 dB
l00 watts= -6 dB
2,500,000 watts = +38 dB

Note: A reference of 1 milliwatts is used for dBm's
1 milliwatts = 10 log(1 / 1) = 0 dBm
5 milliwatts = 10 log(5 / 1) = +7 dBm
500 milliwatts = +27 dBm
0.001 milliwatts = -30 dBm

For Voltage, Power = IE = (E/R)E = E^2/R

To express a Voltage Ratio in dB's, use dB = 10 log($Power_2$ / $Power_1$) = 10 log[(E_2^2/R) / E_1^2/R)]
10 log[(E_2^2/R) / E_1^2/R)] = 10 log(E_2^2 / E_1^2) = 20 log(E_2 / E_1)

For Power Ratio dB = +3, 20 log(E_2 / E_1) = +3
For Power Ratio dB = -3, 20 log(E_2 / E_1) = -3
For Power Ratio db = 0, 20 log(E_2 / E_1) = -0.15 and E_2 / E_1 = 0.707 = SQRT(2) / 2

Equations and Relationships

Inductive Reactance $X_L = 2\pi f L$

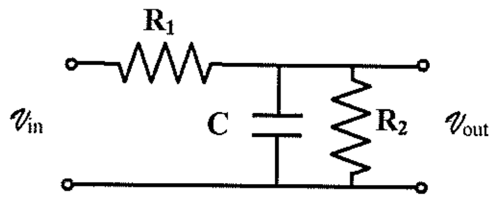
Capacitive Reactance $X_C = \frac{1}{2\pi f C}$

	RC Circuit	RL Circuit	RCL Circuit
Cut-off Frequency	$f_0 = \frac{1}{2\pi\ RC}$	$f_0 = \frac{1}{2\pi\ L/R}$	$f_0 = \frac{1}{2\pi\sqrt{LC}}$
Time Constant	$t = RC$	$t = L/R$	$t = \frac{R\sqrt{C/L}}{2}$

RCL Series Impedance Z = R + jϖL - j(1/ϖC) $|Z| = \sqrt{R^2 + (X_L - X_C)^2}$

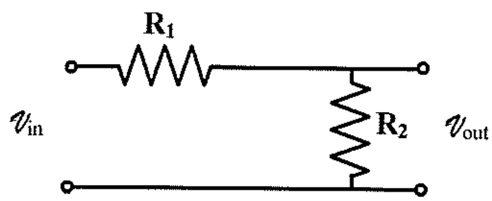
RCL Parallel Impedance Z = R + j(ϖL / (1 - ϖ²LC)) $|Z| = \sqrt{R^2 + \left(\frac{X_L X_C}{X_L - X_C}\right)^2}$

Common Configuration

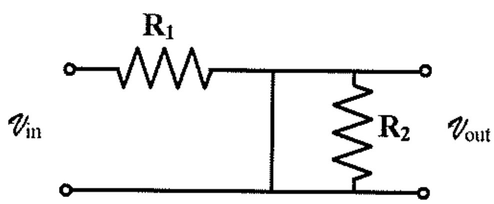


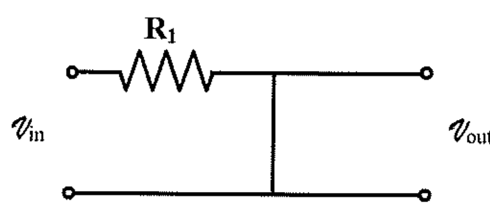
Notes:

When $\omega = 0$, $X_C \rightarrow \infty$, i.e., **C** appears as an open circuit, so that $V_{out} = \frac{R_2}{R_1 + R_2}$



When $\omega >> 0$, $X_C = 0$, i.e., **C** appears as a short circuit, so that $V_{out} = 0$





Passive Filter Characteristics and Terminology