REGISTRATION OF ROTATED IMAGES BY INARIANT MOMENTS

A. Goshtasby
Dept. of Computer Science
Michigan State University
E. Lansing, Michigan 48824

and

W. R. Enslin
Center for Remote Sensing
Michigan State University
E. Lansing, Michigan 48824

Abstract: A new technique for registration of rotated images using invariant moments as the similarity measure is given. If circular windows with invariant moments are used, then window search can be carried out on rotated images in the same manner cross-correlation with rectangular windows is used on translated images. Normalization of invariant moments for more accurate window similarity measurement is also given. The proposed approach is tested against two images from different satellites that have translational and rotational differences.

1. INTRODUCTION

Image registration is the process of determining the position of corresponding points in two images of the same scene. Usually image registration involves resampling of one of the images so that a point in the scene has the same coordinate values in both of the images. In this paper registration of images that have translational and rotational differences will be discussed.

Image registration has many applications. Some of the applications of image registration are in change detection, motion analysis, object tracking, and object recognition. Analysis of two or more images of the same scene often requires registration of the images.

One of the first attempts to register images digitally was made by Anuta [Anuta 69]. He used cross-correlation as the similarity measure to search for corresponding windows in the two images. The centers of the resultant matched windows were taken as corresponding control points for estimating the translational difference between the images.

Image registration by cross-correlation has proven to be very effective and it is still one of the best techniques in image registration. However, this technique can only register images that have translational differences. Computation of cross-correlation is also time consuming. To speed up the process, Anuta used the fast Fourier transform algorithm to compute the cross-correlation values [Anuta 70]. Sum of absolute differences has also been used as an alternative similarity measure to speed-up the window search process [Barnea 72].

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Most of the work in image translational registration has been centered on speed-up techniques rather than the accuracy of the registration. Dewdney has proposed a steepest-descent algorithm to limit the window search domain and thus achieve a higher speed [Dewdney 78].

A two-stage window search technique has also been used [Vanderburg 77]. In this technique, first a subwindow is used to locate possible locations for a match. Then the whole window is used to locate the best match position among the possible ones. This technique was later extended to the coarse-fine searching technique, where a coarse window is used first followed by a finer resolution window [Rosenfeld 77]. This window search process has been expanded to the multi-stage search technique in which in the first stage, the lowest resolution window is used and then higher resolution windows are successively used in each subsequent stage [Hall 76, Tanimoto 81]. Other similarity measures which have been used in the window search process are Haar transform coefficients [Chandra 82], Walsh-Hadamard transform coefficients [Schutte 60], and invariant moments [Wong 78].

When the images are rotated with respect to each other, the above window search techniques for determination of corresponding points in images will fail. This is because, even though the centers of two windows correspond, we may obtain low similarity measures due to the fact that other points in the windows may not correspond to each other. For example, Figure 1 shows two windows in which their centers correspond to each other but since other points in the two windows do not correspond, low similarity is obtained.

As a matter of fact, similarity measure is not the only problem. When two images are rotated with respect to each other, it is impossible for two rectangular windows to contain the same parts of the scene (except when the two windows are rotated by a multiple of 90 degrees with respect to each other). As shown in the two rotated windows of Figure 1, although the centers of the windows correspond to each other they don’t contain the same parts of the scene.

This problem can be alleviated if we use circular windows. Then when the centers of two windows correspond to each other, the two windows will cover the same parts of the scene no matter what the rotational differences are between the windows. Figure 2 shows two circular windows which are rotated with respect to each other and since their centers correspond to each other, they contain the same parts of the scene.

In this paper we will show that rotated images can be searched and matched by using circular windows and invariant moments as the similarity measure. Hu has derived a set of invariant moments such that if two windows contain the same pattern, they will have similar invariant moments no matter what their rotational differences are [Hu 62]. We will normalize the invariant moments and use them in the computation of the similarity measure between two windows.

In the following, invariant moments are first described and their use in template matching is given. Then the window search process using invariant moments with circular windows is discussed including a procedure for the determination of registration parameters. Finally, the result of the proposed approach on registration of multi-satellite images is presented.

2. IN Variant MOMENTS

The two dimensional \((p+q)\)th order moment of a digital image \(f\) is defined by

\[
m_{pq} = \sum_{x} \sum_{y} x^p y^q f(x,y)
\]  

(1)
where $f(x,y)$ is the pixel value of image $f$ at location $(x,y)$ [Hu 62].

Measure $m_{pq}$ changes if image $f$ is translated. To make $m_{pq}$ invariant with respect to translation of $f$, $m_{pq}$ is modified as below,

$$u_{pq} = \bar{x}f(x', y')^2f(x,y)$$  \hspace{1cm} (2)

where

$$x' = \frac{\bar{x}xf(x,y)}{\bar{x}f(x,y)} = m_{10}/m_{00}$$
$$y' = \frac{\bar{x}yf(x,y)}{\bar{x}f(x,y)} = m_{01}/m_{00}$$

$u_{pq}$ is called the $(p+q)$th order central moment of image $f$ and is invariant with respect to translation of image $f$ [Hu 62].

We still cannot apply $u_{pq}$ in the template search process because $u_{pq}$ varies with respect to the rotation of image $f$. Hu has been able to derive moments that are invariant with respect to rotation [Hu 62]. He has shown that an infinite number of such moments exist. The second-order and third-order moments which are invariant with respect to rotation (and translation) of $f$ are given by,

$$a_1 = u_{20} + u_{02}$$  \hspace{1cm} (3)
$$a_2 = (u_{20} - u_{02})^2 + 4u_{11}$$  \hspace{1cm} (4)
$$a_3 = (u_{30} - 3u_{12})^2 + (3u_{21} - u_{03})^2$$  \hspace{1cm} (5)
$$a_4 = (u_{30} + u_{12})^2 + (u_{21} + u_{03})^2$$  \hspace{1cm} (6)
$$a_5 = (u_{30} - 3u_{12})(u_{30} + u_{12})(u_{30} + u_{12})^2 - 3(u_{21} + u_{03})^2 + (3u_{21} - u_{03})^2$$  \hspace{1cm} (7)
$$a_6 = (u_{20} - u_{02})(u_{30} + u_{12})^2 - (u_{21} + u_{03})^2 + 4u_{11}(u_{30} + u_{12})(u_{21} + u_{03})$$  \hspace{1cm} (8)
$$a_7 = (u_{30} - 3u_{12})(u_{30} + u_{12})(u_{30} + u_{12})^2 - 3(u_{21} + u_{03})^2 - (3u_{21} - u_{03})^2$$  \hspace{1cm} (9)

Usually more than one moment is used in decision making. One way to measure similarity between two windows is to compute the distance between two vectors of moments. The smaller the distance, the more similar the two windows [Giuliano 61]. This method however requires that the feature elements in the vectors be of the same unit (or scale). Moments of different orders do not have the same scale. Table 1.a shows a 16x16 window whose moments up to order 5 are shown in Table 1.b. As can be seen, moments of different order are in different magnitudes because of scale differences.

The correlation of the logarithm of moments has been used by Wong to measure the similarity between two windows [Wong 78]. Again, the moments of different orders have different scales and using the logarithm of the moments does not solve the problem. Note that in Table 1.b, the logarithm of the moments increase with the order of the moment. If the correlation of logarithm of moments is determined, high correlation values will always be obtained regardless of the dissimilarity between the two windows. So, the correlation of the moments is not an appropriate measure of similarity either. The correlation between two sets of numbers is defined when the numbers in each set
are in the same scale. When features of different scales are used in the computation of correlation value, the feature with the largest scale dominates the correlation value.

Garrett has proposed the concept of pairing function where the similarity between two sets of features is determined by quantizing the features and counting the number of quantized features that are equal [Garrett 76]. In this technique, since the features in the two sets are quantized into the same number of levels, it is as if the features in the two sets are in the same scale. This alleviates the problem of scale differences but this similarity measure does not provide information about how closely two features match. When the feature values are quantized, critical information needed in similarity measurement might disappear. So, the pairing function is also not appropriate for measuring the similarity between two windows when moments are used as features.

To overcome the above problems, we normalize the moments so that they all have the same scale. The normalized moment of order \((p,q)\) for image \(f\) with dimensions \(N\) by \(N\) is defined by,

\[
M_{pq} = \sum_{x=1}^{N} \sum_{y=1}^{N} (x/N)^p (y/N)^q f(x,y)
\]

where

\[
x^* = (1/N) \sum_{x=1}^{N} x - N(N+1)/2N - (N-1)/2
\]

\[
y^* = (1/N) \sum_{y=1}^{N} y - (N+1)/2
\]

So, actually

\[
M_{pq} = \sum_{x=1}^{N} \sum_{y=1}^{N} \left(\frac{x}{(N+1)/2}\right)^p \left(\frac{y}{(N+1)/2}\right)^q f(x,y)
\]

\[
= \left(\frac{2}{(N+1)}\right)^p \left(\frac{2}{(N+1)}\right)^q \sum_{x=1}^{N} \sum_{y=1}^{N} x^p y^q f(x,y)
\]

So, the normalized \((p,q)\)th order moment is equal to the \((p,q)\)th order moment multiplied by the scaling factor \(\left(\frac{2}{(N+1)}\right)^p \left(\frac{2}{(N+1)}\right)^q\). Table 1.b shows the normalized moments for the window of Table 1.a. As can be seen, moments of all orders have the same scale now.

To determine the normalized central moments, we replace \(x\) by \(x/(N+1)/2\) and \(y\) by \(y/(N+1)/2\) in formula (2).

\[
U_{pq} = \sum_{x=1}^{N} \sum_{y=1}^{N} [(x-x^*)/((N+1)/2)]^p [(y-y^*)/((N+1)/2)]^q f(x,y)
\]

\[
= \left(\frac{2}{(N+1)}\right)^p \left(\frac{2}{(N+1)}\right)^q \sum_{x=1}^{N} \sum_{y=1}^{N} (x-x^*)^p (y-y^*)^q f(x,y)
\]

In the same manner we can determine the normalized invariant moments (formulas (3) to (9)), in which each is multiplied by only one scaling factor. We will be using the normalized invariant moments from two circular windows as features and by computing the cross-correlation between these feature values, the similarity of windows will be determined.

3. DETERMINATION OF REGISTRATION PARAMETERS

The ultimate goal is to determine the translational and rotational differences (registration parameters) between two images. A minimum of two pairs of corresponding points from the two images are required to determine the registration parameters. To determine a pair of corresponding points in two
images, a small window called a template, is taken from one image and a larger window, called a search area, is extracted from the other image. The template is shifted in the search area until the position where the template best matches in the search area is determined. At the best match position, the center of the template corresponds to the center of the window in search area over which the template lies.

In order to make the window-matching process more reliable, templates should be taken in a high variance (high information) area in the image. An automatic procedure for selection of high information templates has been defined [Davis 78]. This procedure selects windows that are well separated and have a large number of high gradient, high curvature, connected edges in the image. The search area should be selected such that the window corresponding to the template is not missed.

Given a circular template and a search area, the following algorithm determines the position where the template best matches in the search area using invariant moments.

Algorithm A: Template matching by invariant moments.

1. Select template $W$ of size $N \times N$ from a high information area in one image.
2. Select search area $S$ of size $M \times M$ in the other image such that it contains the template.
3. Determine the $n$ normalized invariant moments of the template, let them be $a_1, a_2, \ldots, a_n$.
4. For $i = 0$ to $M-N/2$+1
5. For $j = N/2$ to $M-N/2$+1
6. Determine the $n$ normalized invariant moments of the window cantered at $(i,j)$ in the search area and size $N \times N$, let them be $a'_1, a'_2, \ldots, a'_n$.
7. Find the cross-correlation between $\{a_1, a_2, \ldots, a_n\}$ and $\{a'_1, a'_2, \ldots, a'_n\}$, let it be $r(i,j)$.
8. Let $(i', j')$ be that $(i,j)$ such that $r(i', j') = \max_r(i,j)$. Then $(i', j')$ would be the coordinates of the center of the window in the search area which best matches with the template.
9. The center of the template corresponds to location $(i', j')$ in the search area. By this, a pair of corresponding points from the two images are found.

Template size $N \times N$, search area size $M \times M$, and the number of invariant moments $n$ are specified by the user. If the template size is too small, it may not contain enough information to carry out a reliable search. If the template size is too large, the search process becomes costly. So, selecting the right template size is very important.

Usually, smooth areas of an image require larger template sizes than detailed ones. Since an image may contain both smooth and detailed areas, a constant template size may not be adequate and different size templates may have to be used. Dynamically determining template size at every step, however, is time consuming and it adds overhead to the already slow search process.

Traditionally, template size has been selected heuristically. We have followed the tradition and have tried different template sizes. The smallest size which gave reasonable accuracy was taken as the template size. For the images we had, templates of size 16x16 turned out to be appropriate.

Search area size is another parameter which should be given by the user and it should be taken in such a way that it contains the template. Another parameter is the number of invariant moments required in the matching process. The greater the number of moments the more accurate the matching but the slower
the search process. We have used 8 normalized invariant moments. These are
the zeroth order, second order, and third order normalized invariant moments.
The zeroth order moment is equal to the sum of intensity values in a window.
\[ a_0 = \mu_{00} = \sum f(x, y) \]

Using Algorithm A, a number of corresponding points from the two images can
be obtained. There are possibilities of mismatches in the process and some of
the corresponding points may not actually correspond to each other. Non-
corresponding points should be eliminated before registration parameters are
computed.

An effective method for detection of points that do not correspond to each
other is given in [Fischler 81]. In this method, for each two pairs of

\[
x' = x \cos \theta - y \sin \theta + h
\]
\[
y' = x \sin \theta + y \cos \theta + k
\]

where \((x', y')\) and \((x, y)\) are the coordinates of corresponding points in the two
images. If we replace \(\cos \theta\) by \(a\) and \(\sin \theta\) by \(b\) we get,
\[ x' = ax - by + h \]
\[ y' = bx + ay + k \]

Now if coordinates of \(n\) corresponding points from the two images are given, we
can estimate \(a, b, h,\) and \(k\) by minimizing the following sum of squared errors.
\[ E = \frac{1}{n} \sum [(x_i' - (ax_i + by_i + h))^2 + (y_i' - (bx_i + ay_i + k))^2] \]

To minimize \(E\) with respect to \(a, b, h,\) and \(k,\) we find the partial
derivatives of \(E\) with respect to \(a, b, h,\) and \(k.\) Set them equal to zero, and
solve the system of linear equations.
\[
\begin{bmatrix}
\Sigma(x_i^2 + y_i^2) & 0 & \Sigma x_i & \Sigma y_i \\
0 & \Sigma(x_i^2 + y_i^2) & \Sigma y_i & -\Sigma x_i \\
\Sigma x_i & \Sigma y_i & n & 0 \\
\Sigma y_i & -\Sigma x_i & 0 & n
\end{bmatrix}
\begin{bmatrix}
a \\
b \\
h \\
k
\end{bmatrix}
= 
\begin{bmatrix}
\Sigma(x_i'y_i + y_i'x_i) \\
\Sigma(y_i'x_i - x_i'y_i) \\
\Sigma x_i' \\
\Sigma y_i'
\end{bmatrix}
\]

where \((h, k)\) is the translational difference and \(\theta = \cos^{-1}a\) and \(\theta = \sin^{-1}b\) is the
rotational difference between the images. Knowing \(h, k,\) and \(\theta,\) we can map one
image into another by the transformation functions of (10).

4. RESULTS

Two images from two different satellites were used to test the above image
registration technique. Figure 3 shows a thermal-infrared image of Michigan
obtained by the Geostationary Operational Environment Satellite (GOES) on 24 June 79 at 3:00 PM. Figure 4 is a day-visible image of approximately the same area obtained by the Heat Capacity Mapping Mission (HCMM) satellite on 26 Sept. 79. Both GOES and HCMM images were already geometrically corrected. The original HCMM image has 500x500 meter resolution. Since the GOES image has 8x11 km resolution, the HCMM image was smoothed in 16x22 neighborhoods and was resampled to the same resolution as the GOES image. The two images now have the same scale but have translational and rotational differences.

Since two windows with similar pattern but different intensity values result in different invariant moments, we have used the gradient of the two images rather than the images themselves in the registration. This reduces the effect of intensity difference between the two images.

Six points from the GOES image was selected by hand. Circular templates with an 8 pixel radius were taken centered at the selected points. Search areas with a radius 16 pixels were selected from the HCMM image in such a way that they included the template areas. For each template and search area, Algorithm A was executed to determine the best match position of the template in the search area.

Transformation parameters were obtained by using the 6 pairs of corresponding points from the two images and solving the system of linear equations shown by (11). The rotational difference between the two images was 11.5 degrees and the translational difference between the two images was (9.4,-13.8). The GOES image was resampled by the nearest neighbor method and using the transformation,

\[ x' = 0.98x - 0.20y + 9.4 \]
\[ y' = 0.20x + 0.98y - 13.8 \]

where \((x',y')\) and \((x,y)\) are coordinates of corresponding points in the HCMM and GOES images, respectively. Figure 5 shows the resampled GOES image. GOES and HCMM images are subtracted in Figure 6 to visualize the registration.

5. CONCLUSION

A technique for registration of rotated images using normalized invariant moments was given. The technique first locates a number of corresponding points in two images by window searching and then estimates the registration parameters using the corresponding points and minimizing the sum of squared errors.

This technique can be used in registration of multi-satellite and multi-temporal images. If the images have scaling differences, a low-pass filter can be used on the high resolution image and resample it to the same resolution as the low resolution image since resolution of satellite images are known. This alleviates the problem of scale difference between the images. The technique given above can then be used to determine the rotational and translational differences between the images. An example was given showing registration of HCMM and GOES images which had known scaling difference but unknown rotational and translational differences.

REFERENCES

[Anuta 69] "Digital Registration of Multi-Spectral Video Imagery", Paul E. 1039


Table 1. (a) A window and (b) its moments, logarithm moments, and normalized moments.

Figure 1. Two rotated square windows with the same center point.
Figure 2. Two rotated circular windows with the same center point.

Figure 3. GOES thermal IR image of Michigan (24 Jun 79).

Figure 4. HCMM day-visible image of Michigan (26 Sep 79).

Figure 5. GOES image resampled to overlay with the HCMM image.

Figure 6. GOES and HCMM image difference.