A High-Frequency Multipeak Model for Wide-Angle SAR Imagery

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Abstract—A new modeling method for representing distributed scattering centers in wide-angle synthetic aperture radar (SAR) is presented. The proposed multipeak model approximates amplitudes of localized image peaks that typically appear at a single pixel or as an in-line set of pixels in a SAR image. In this way, the multipeak model is an improvement over existing peak models which poorly represent distributed canonical scatterers, such as the common dihedral with a fold line oriented parallel to the imaging plane. The model is derived from a wide-angle approximation of the well-known attributed scattering center or parametric models when under the action of a linear imaging operator. It is shown that, under typical imaging conditions of $10^\circ$ or more in the synthetic aperture, the multipeak model approximates the image peak amplitudes due to distributed canonical scatterers as if they are due to an equivalent point scatterer with an azimuth-independent dispersive amplitude function in the spectral domain. This improves parameter estimation and scatterer classification, and it is also shown that the imaging relative error due to the approximation is less than 2% for other common image processing conditions such as tapered windowing in azimuth and when the canonical scatterer is at least ten wavelengths in size. A distinct advantage of the multipeak model over point scatterer models is that parameter estimation and scatterer classification can be performed solely in the spatial domain on a pixel-by-pixel basis and efficiently integrated within a linear SAR imaging process. To illustrate the benefits and limitations of the approach, parameter estimation and scatterer classification experiments are presented using simulated SAR data.

Index Terms—Distributed scatterer model, joint frequency-polarization scatter classification, parametric scattering model, synthetic aperture radar (SAR), wide-angle SAR.

I. INTRODUCTION

SYNTHETIC aperture radar (SAR) images contain bright spots representing locations where strong backscatterers are present in the scene. Within these bright spots are pixels having localized peak intensities, called image peaks. For example, Fig. 1 shows a SAR image of a residential scene taken from the moving and stationary target acquisition and recognition data set, where bright spots and image peaks are noticeable throughout the image [1], [2].

SAR signals have frequency and azimuth diversity, and in the case of polarimetric SAR, they have polarimetric diversity as well [3]. Therefore, by analyzing the dispersive, anisotropic, and polarimetric (DAP) characteristics of bright spots and peaks, it is possible to measure the likelihood that a bright spot corresponds to a specific type of object. Thus, one may ask: Are bright spots in ovals A and B more likely due to automobiles or construction equipment or is it possible to determine if the bright line in oval C is more likely due to a fence or a pipeline?

In general, the number of possible inquiries is unlimited, and reasonable answers depend upon contextual clues which are best deduced by human operators. Therefore, it is desirable to succinctly present the DAP characteristics of bright spots and peaks within the context of the image. Because SAR signals are processed in both the spatial and spectral domains, it is helpful to categorize dispersive and anisotropic analysis methods into two categories—image segmentation and phase history decomposition—according to the domain in which they decompose the signal. (The most common polarimetric analyses are performed in the spatial domain.)

Image segmentation methods decompose the SAR signal in the spatial domain. The segments are localized to either bright spots or known locations of interest. A time–frequency transform is applied to each segment to produce a coarse-resolution spectrum for further analysis. Image segmentation methods are best represented by two research projects: the hyperimage concept developed at ONERA, the French Aerospace Laboratory [4]–[6], and parametric scatterer classification developed at The Ohio State University [7]–[10]. Other image segmentation methods in the literature are generally a variation of these two primary methods.

The hyperimage concept simply displays the 2-D resultant of a time–frequency transform. Unfortunately, a human analyst must manually select the image segments and transforms and then scrutinize each display outside of the image context. Thus, the hyperimage concept is best suited for highly trained analysts having ample time to conduct the analyses.
Parametric scatterer classification matches the image segments and their spectra to the spectral and polarimetric responses expected for a set of ideal canonical scatterers. Examples of canonical scatterers include trihedral, dihedral, plates, cylinders, and spheres. The set of canonical scatterers is limited, so that classification results can be succinctly presented as an overlay within the context of the image. The classification results assume that bright spots in the scene are due to geometric objects with an amplitude that responds to changes in frequency, azimuth, and polarimetry in accordance with well-known physical models of electromagnetic scattering [11]. Thus, classification accuracy can suffer when canonical scatterers are poorly isolated in the SAR image or when noncanonical scatterers, such as resonant cavities, are present. Therefore, in practice, image segmentation still requires human supervision so that only isolated canonical scatterers are considered.

Phase history decomposition methods decompose the SAR signal in the spectral domain. The phase history is subdivided by regular intervals into subdomains. Coarse-resolution subimages are reconstructed from each subdomain for further analysis. In practice, frequency bandwidth is often more restricted than angular bandwidth; therefore, the most common decompositions are in azimuth. The full aperture is subdivided into two or more subapertures of equal width. The resulting subimages are diverse in azimuth but with coarser resolution in cross range than a full-aperture image. These subimages reveal the anisotropic scattering behavior of each image pixel, which can be analyzed or, in the case of multilook SAR, averaged over the full aperture. Similarly, for decompositions in frequency, the bandwidth is subdivided into two or more subbands. Because the available bandwidth is often limited, the subbands are usually equal to half of the full bandwidth. The resulting subimages are diverse in frequency but with coarser resolution in range than a fullband image. These subimages reveal the dispersive scattering behavior of each image pixel, which can be analyzed or directly displayed in tricolor [6]. Finally, note that azimuth and frequency decompositions are easily combined and that the polarimetric analyses commonly performed on images are also valid for use with subimages.

By employing domain decomposition techniques, the coarse-resolution subimages can be interpolated and summed to closely approximate a fine-resolution image reconstructed from the full-aperture fullband SAR signal. Therefore, phase history decomposition methods are computationally efficient. In addition, the DAP characteristics of bright spots or peaks can be automatically extracted from the subimages and succinctly presented within the image context without the need for image segmentation, spectral analysis, or, most importantly, human supervision. Because of these advantages and the fact that canonical scatterers often comprise objects of interest, it is desirable to develop a model to predict the intensity of subimage peaks due to canonical scatterers. Such models are called peak models.

For SAR signals, spatial resolution is inversely proportional to spectral bandwidth. This is a manifestation of the Gabor limit [12]. Therefore, phase history decomposition methods essentially operate by sacrificing spatial resolution information to obtain spectral bandwidth information. Typically, the spectral diversity of the subimages is limited in order to maintain reasonable precision for scatterer localization. As a result, only slowly varying pixel intensities can be accurately measured by subimage analysis.

All canonical scatterers have a slowly varying amplitude response to changes in frequency, but only canonical point scatterers have a slowly varying amplitude response to changes in azimuth. Canonical point scatterers, such as a dihedral, are characterized by the fact that their peak intensities are located at a single location, or point, in the image, similar to the peak of a digitized sinc function [13]–[21]. In contrast, distributed canonical scatterers have a rapidly varying sinclike amplitude response in azimuth [7], [21]. Distributed canonical scatterers, such as the common dihedral with a fold line oriented parallel to the imaging plane, are characterized by the fact that their energy usually spreads across multiple pixels. This is representative of a digitized rectangular function with a region of support over multiple samples and ripple in accordance with the Gibbs phenomenon [22]. Thus, distributed canonical scatterers often appear as a set of in-line peaks of approximately the same amplitude. For example, in Fig. 1, oval C shows a likely example of a distributed canonical scatterer, while ovals A and B show likely examples of canonical point scatterers.

Peak models have already been developed to predict the intensity of subimage peaks due to canonical point scatterers [20], [23]–[27]. However, to our knowledge, no model exists to predict the intensity of subimage peaks due to distributed canonical scatterers. This paper presents a multipixel model to approximate the amplitudes of localized image peaks that typically appear at a single pixel or as an in-line group of pixels in a SAR image. The multipixel model is derived from a wide-angle approximation of the well-known parametric model for canonical scatterers, which, in turn, is based on physical models of electromagnetic scattering [7]. The multipixel model is an improvement over existing peak models because it explicitly accounts for distributed canonical scatterers by replacing the rapidly varying azimuth dependence of the amplitude function in the spectral domain with a modified slowly varying frequency dependence. In this way, the multipixel model accounts for distributed canonical scatterers while retaining all of the advantages associated with phase history decomposition, such as efficient parameter estimation solely in the spatial domain. In addition, because canonical scatterers with tilt angles near 0° behave as distributed canonical scatterers and those with tilt angles near 90° behave as canonical point scatterers, another benefit of the multipixel model is that, for wide-angle SAR imagery, canonical scatterer tilt angles near 0° and near 90° can be discriminated without the need for fully polarimetric SAR data.

This paper is organized as follows. Section II explains how a SAR phase history is modeled as a sum of phase histories due to multiple scattering centers. It presents the hypothesis which motivates the multipixel model that, under a high-frequency wide-angle assumption, the peak amplitude due to a distributed canonical scatterer can be approximated by the peak amplitude due to an equivalent canonical point scatterer. Section III presents the multipixel model. It describes how and under what conditions the imaging operator integrates the sinclike amplitude response in azimuth to produce a pointlike amplitude response for distributed canonical scatterers. Numerical analysis reveals that, for typical imaging algorithms, the error due to the approximation is 2% or less when canonical scatterers are ten wavelengths long or longer and aperture widths are 10° wide or
wider. Asymptotic analysis further reveals that the model error is best controlled with the use of a tapered window in azimuth, such as the raised-cosine windows typically employed in SAR imaging algorithms. Section IV presents a simple parameter estimation and canonical scatterer classification algorithm based on the multipeak model. Experimental results are obtained from simulated data using a wide variety of isolated canonical scatterers and varying aperture widths. Additional results are shown on the multipeak model. Experimental results are obtained from simulated data using a wide variety of isolated canonical scatterers and varying aperture widths. Additional results are shown from simulated data using a wide variety of isolated canonical scatterers and varying aperture widths. Additional results are shown.

II. PROBLEM STATEMENT

A. Radar Imaging

Typically, the phase history $\tilde{G}$ in a 2-D SAR system is displayed as a manifold in the spectral domain with a finite region of support as shown in Fig. 2. The samples recorded in frequency and azimuth produce a phase history in polar coordinates, where $f = \sqrt{f_x^2 + f_y^2}$ and $\theta = \tan^{-1}(f_y/f_x)$. This follows the convention for tomographic reconstruction where the axes $f_x$ and $f_y$ result from the 2-D Fourier transform of the image to be reconstructed on axes $x$ and $y$, respectively [3]. The remaining discussion assumes that the sampling rate is sufficient to prevent aliasing in the image and that all amplitude variations due to antenna gain pattern and spherical wave propagation are normalized between samples. The image $\tilde{g}$ provides an estimate of the reflectivity of the scene and is reconstructed using an appropriate transformation from the spectral domain to the spatial domain. This transformation can be succinctly represented by [3]

$$\tilde{g}(x, y) = B\{\tilde{G}(f, \theta)\}$$  \hspace{1cm}  (1)$$

where the conventional 2-D imaging operator is

$$B\{\cdot\} = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \cdot e^{j2kr} |f| df d\theta$$  \hspace{1cm}  (2)$$

where $k = 2\pi f/c$ is the wavenumber and $r(x, y, \theta)$ is the distance from the radar phase center to the spatial coordinates in the imaging plane as a function of azimuth angle.

Furthermore, $\tilde{G}$ is defined over finite regions of support $f \in \{f_c - (B/2), f_c + (B/2)\}$ and $\theta \in [\theta_c - (\Theta/2), \theta_c + (\Theta/2)]$, where $f_c$ is the center frequency of the phase history with bandwidth $B$ and $\theta_c$ is the center angle of the phase history with aperture width $\Theta$. In this case, the finite regions of support can be represented by a band-limited filter or window in frequency $H_B(f - f_c)$ and an aperture-limited filter or window in azimuth $H_{\Theta}(\theta - \theta_c)$. Hence, the image is reconstructed from the windowed phase history as

$$\tilde{g}(x, y) \approx B\{H_B(f - f_c)H_{\Theta}(\theta - \theta_c)\tilde{G}(f, \theta)\}$$  \hspace{1cm}  (3)$$

where the choice of $H_B$ and $H_{\Theta}$ depends on the need for image resolution versus image contrast, and for convenience, the dependence of $\tilde{g}$ on these functions is suppressed. Additionally, if multiple polarization channels are available, the transformation is performed for the phase history associated with each channel.

B. Sum of Scattering Centers

By linear superposition, a SAR phase history can be modeled as a sum of phase histories due to multiple scattering centers expressed as

$$\tilde{G}(f, \theta) = \sum_{q=1}^{Q} \tilde{S}_q(f, \theta)e^{-j2kq}$$  \hspace{1cm}  (4)$$

where $\tilde{S}_q(f, \theta)$ is the amplitude function and $r_q(x_q, y_q, \theta)$ is the distance from the antenna phase center to the $q$th scattering center as a function of azimuth angle. Likewise, the SAR image can be modeled as a sum of images due to multiple scattering centers expressed as

$$\tilde{g}(x, y) = \sum_{q=1}^{Q} \tilde{s}_q(x, y)$$  \hspace{1cm}  (5)$$

where $\tilde{s}_q$ is the resultant of the imaging operator acting on the $q$th windowed phase history in (4).

Conventionally, the scattering centers are modeled as ideal point scatterers. For an ideal point scatterer, the amplitude function is a constant, and its image is called the point spread function. Conventional imaging operators, such as the one described in Section II-A, are designed to maximize the response of ideal point scatterers. Unfortunately, the number of point scatterers required to accurately model or simulate a SAR phase history is typically quite large.

A parsimonious sum is possible when the scattering centers are modeled as canonical scatterers [7]. For canonical scatterers, the amplitude function is parameterized by physical models of electromagnetic scattering based on geometric optics (GO) and the geometric theory of diffraction (GTD) [28]. A restricted set of possible geometrical shapes, combined with the high-frequency far-field assumptions in GO/GTD, produces a model with only a few parameters. The model can be used to simulate a SAR phase history, or the parameters can be estimated from SAR data to detect or classify canonical scatterers in the scene.

C. Hypothesis

The hypothesis of this paper is that the SAR image peak amplitude due to a distributed canonical scatterer can be modeled as an equivalent canonical point scatterer having an azimuth-independent scaled amplitude function in the spectral domain and a frequency dependence of reduced order. This is expressed as

$$|\tilde{s}_q(x_q, y_q)| = \left| B\left\{H_BH_{\Theta}\tilde{S}_q(f; w_q)\right\}(x_q, y_q) \right| \approx \left| B\left\{H_BH_{\Theta}\tilde{S}_q(f; w'_q)\right\}(x_q, y_q) \right|$$  \hspace{1cm}  (6)$$
where $\mathbf{w}_q$ is the set of parameters describing the amplitude function of the $q$th distributed canonical scatterer (e.g., scatterer length and orientation) while $\mathbf{w}_0$ is a reduced set of parameters describing the amplitude function of the equivalent canonical point scatterer. Here, $H_H$ and $H_B$ are short-hand notations for $H_B(f - f_c)$ and $H_B(\theta - \theta_c)$, respectively. The approximation is suitable for canonical scatterers of sufficient electrical length and apertures of sufficient width. These are the high-frequency and wide-angle assumptions of the multiplet model. The following section derives the set of parameters for the canonical model and discusses the error due to the approximation, as well as the conditions for which the error is well controlled.

### III. Model Development

#### A. Canonical Scattering Model

For a copolarized channel, the amplitude response of the electric field backscattered from canonical scatterers has a frequency response predicted by GO and GTD as [7], [28]

$$\tilde{S}_f(f; A, \alpha) = A(jf)^{\frac{\alpha}{2}}$$

(7)

where $A$ is a complex-valued amplitude related to the physical size of the canonical scatterer and $\alpha$ is an integer value depending upon the local curvature of the canonical scatterer’s shape. Note that the GO/GTD-based model in (7) assumes canonical scatterers that are perfect electrical conductors. The values of $\alpha$ for common shapes are well known and listed in Table I. However, there is clearly an ambiguity when classifying a single geometry by estimating $\alpha$ only.

At a single azimuth angle or over an extremely narrow aperture, the amplitude responses for all canonical scatterers are well modeled by (7). However, for typical SAR apertures, the amplitude response of the backscattered field for distributed canonical scatterers has an azimuth dependence dominated by a sinc-like pattern. Common distributed canonical scatterers include the following:

1) flat plates at the broadside aspect;
2) dihedrals with fold lines parallel to the azimuth direction;
3) cylinders with axis of rotation parallel to the azimuth direction;
4) edges or wires lying parallel to the azimuth direction.

Of these, the dihedral, in particular, is often present in SAR imagery of man-made structures. For example, the side of a building and the ground form a dihedral with a fold line often lying parallel to the azimuth direction. The amplitude response as a function of azimuth is well modeled by [7], [29]

$$S_0(f, \theta; L, \theta_0) = \text{sinc}\left(\frac{2}{c}fL \sin(\theta - \theta_0)\right)$$

(8)

where the sinc function is defined as $\text{sinc}(t) = \sin(\pi t)/\pi t$, $L$ is the effective length of the canonical scatterer as projected onto the slant plane, and $\theta_0$ is the orientation angle normal to this projection and referenced to the center angle of the aperture [7]. The dependence upon $\theta_c$ is suppressed in the notation because the simplification $\theta_c = 0$ can often be made without loss of generality. An example of the effective length and orientation angle for a cylinder is shown in Fig. 3.

Using (7) and (8) to model the amplitude response, the phase history due to the $q$th canonical scatterer is

$$\tilde{S}_q(f, \theta; \mathbf{w}_q) = \tilde{S}_f(f; A_q, \alpha_q) \times S_0(f, \theta; L_q, \theta_0) e^{-j2k\tau}$$

(9)

where $L_q > 0$ for a distributed canonical scatterer and $L_q \approx 0$ for a canonical point scatterer, setting $S_0 \approx 1$.

By substituting the canonical scatterer model from (9) into (3) and (2), the image due to the $q$th canonical scatterer is

$$\tilde{s}_q(x, y) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\pi}^{\pi} H_B(f - f_c) H_{\theta}(\theta - \theta_c)$$

$$\times \tilde{S}_f(f; A_q, \alpha_q) S_0(f, \theta; L_q, \theta_0)$$

$$\times e^{j2k(r - r_c)} |f| df d\theta.$$  

(10)

#### B. Peak Model

The peak amplitude of (10) resides at location $(x_q, y_q)$ in the image given by

$$\tilde{s}_q(x_q, y_q) = \frac{1}{2\pi} \int_{-\infty}^{\infty} H_B(f - f_c) \tilde{S}_f(f; A_q, \alpha_q) |f|$$

$$\times \int_{-\pi}^{\pi} H_{\theta}(\theta - \theta_c) S_0(f, \theta; L_q, \theta_0) d\theta d\theta.$$

(11)
Note that the integral in azimuth in (11) represents an inverse Fourier transform which affects the image of canonical point scatterers and distributed canonical scatterers differently. A canonical point scatterer with $L \approx 0$ has a constant or slowly varying amplitude response in azimuth, and the image will contain a single peak in accordance with (11). However, a distributed canonical scatterer with $L \gg 0$ has a sinc-like amplitude function in azimuth, and given a sufficient aperture width, the image will contain a rectangular function with ripple. The ripple is a manifestation of the Gibbs phenomenon [22] and results in one or more image peaks depending upon the length of the canonical scatterer and the dimensions of the image pixels. In this case, (11) predicts the amplitude of the rectangular function, including any peaks caused by ripple. The multipeak model derives its namesake from this effect, where distributed canonical scatterers typically appear as an in-line set of multiple peaks in the image.

Consider a distributed canonical scatterer oriented so that the main lobe of the sinc function is contained within the azimuth window. In this case, the resultant of the inner integral in (11) is dominated by the area under the main lobe of the sinc function and upon the azimuth window to suppress the sidelobes of the sinc function. The window and aperture conditions for which the error is well controlled are best understood and illustrated by numerical and asymptotic analyses.

C. Numerical Analysis

The numerical integration in (12) for a range of frequencies and aperture widths is shown in Fig. 4, for the case of $L = 1$ m and a Hanning window for $H_\Theta$. Fig. 4(a) and (b) reveals that $\hat{A}$ becomes independent of frequency when $\Theta \gtrsim 10^\circ$ and $f \gtrsim 3$ GHz for this case. Fig. 4(c) shows similar results for varying orientation angles $\theta_0$. Because raised-cosine windows, other than Hanning windows, are often used in SAR imaging [3], these were also investigated and were observed to produce results similar to that shown in Fig. 4. The accuracy of the approximation decreases gradually as the canonical scatterer becomes electrically shorter, i.e., as $f$ decreases or as $L$ decreases in units of wavelength.

As the model in (9) is based on GO/GTD, it is most accurate for electrically large scatterers, where $L \gtrsim 10\lambda$. This is sometimes referred to as the physical optics region or the high-frequency approximation for electromagnetic scattering. Accordingly, the numerical integration results in Fig. 4 are truncated at $f_{\text{min}} = 3$ GHz, where the $L = 1$ m canonical scatterer is ten wavelengths long. Therefore, $\Theta_{\text{min}} \approx 10^\circ$ is sufficient to support a high-frequency approximation for electromagnetic scattering, and for the purposes of this paper, apertures greater than $10^\circ$ are considered to constitute wide-angle SAR. In other contexts, the definition for wide-angle SAR is constrained to the condition where bandwidth in frequency is much more limited than bandwidth in azimuth [30]–[32], but (12) is not restricted to this condition.
D. Asymptotic Analysis

Setting the limits of integration in (12) to the region of support for $H_\theta$, i.e., $\theta \in [\theta_e - (\Theta/2), \theta_e + (\Theta/2)]$, with the constraint $(\Theta/2) < (\pi/2) - |\theta_0|$ and setting $\theta_e = 0$ without loss of generality, gives

$$\int_{-\infty}^{\infty} H_\theta(\theta) \text{sinc} \left[ \frac{2}{c} f L \sin(\theta - \theta_0) \right] d\theta \approx \frac{c \hat{A}}{2 f L}. \quad (13)$$

The change of variables $a = 2 f L / c$ and $x = \sin(\theta - \theta_0)$ further simplifies the integral to

$$\hat{A} \approx \int_{-1}^{1} \frac{H_\theta(\theta_0 + \sin^{-1}(x))}{\sqrt{1 - x^2}} a \text{sinc}(ax) dx \quad (14)$$

where the earlier constraint causes the limits of integration to be constrained to the interval $[\sin(\Theta/2 - \theta_0), \sin((\Theta/2) - \theta_0)]$ or, more generally, $x \in [-1, 1]$. As $a$ approaches infinity, the second factor in (14) is a form of Dirac’s delta [33]. Under this limit, the sinc function becomes a sampling function so that the first factor is sampled at $x = 0$, and

$$\lim_{a \to \infty} \hat{A} = \lim_{a \to \infty} \int_{-1}^{1} \frac{H_\theta(\theta_0 + \sin^{-1}(x))}{\sqrt{1 - x^2}} a \text{sinc}(ax) dx \approx H_\theta(\theta_0) \quad (15)$$

where $H_\theta(\theta_0)$ is assumed positive for $|\theta_0| < (\Theta/2)$ and zero if otherwise. Under the limit, $\hat{A}$ equals a constant; therefore, the error introduced by the wide-angle approximation in (12) is bounded. That is, for any typical azimuth window, the error bound diminishes with increasing frequency or, equivalently, as the distributed scatter increases in electrical length.

Bounded error alone is not sufficient to make the wide-angle approximation generally useful for time–frequency analysis of SAR imagery. It is also important that $\hat{A}$ be insensitive to changes in frequency or, equivalently, changes in $a$. This sensitivity is revealed by taking the partial derivative with respect to $a$ of the right-hand side of (14), where a partial derivative equal to zero reveals that $\hat{A}$ is independent of $a$. Thus, noting that $\partial(\text{sinc}(ax))/\partial a = \cos(\pi a x)$, the wide-angle-approximation relative error is defined as

$$e(H_\theta, a, \theta_0) = \left| \frac{\int_{-1}^{1} \frac{H_\theta(\theta_0 + \sin^{-1}(x))}{\sqrt{1 - x^2}} \cos(\pi a x) dx}{H_\theta(\theta_0)} \right| \quad (16)$$

where $e = 0$ indicates that $\hat{A}$ is independent of $a$. The plots of the error are shown in Fig. 5 for the case of $\theta_0 = 0$ and for the cases of $a = 20$ and $a = 40$. The plots compare the errors of a rectangular azimuth window and three other windows commonly used in SAR image processing [3]. All windows are in accordance to Matlab default definitions. The Hanning window appears to perform the best overall, while the Taylor window performance is marginal, and the rectangular window performance is notably poor. Similar results are obtained for larger values of $a$ but with lower overall error, as expected. Also, as expected, the error increases rapidly for an aperture width less than $10^\circ$, regardless of the window type. In addition, the results are similar for varying values of $\theta_0$. Because a rectangular window in azimuth causes much larger error than a tapered window, the multipeak model is recommended for use with tapered windows, such as the family of raised-cosine windows typically employed for SAR imagery.

E. Multipeak Model

Based on the azimuth limitations of the model, the peak amplitude of the $q$th scattering center in (11) is approximated as

$$\hat{s}(x_q, y_q) \approx \frac{1}{2\pi} \int_{-\infty}^{\infty} H_B(f - f_q) A_q(j f) \frac{\sin \pi f}{\pi f} \left| f \right| \left| \frac{c A_q}{2 f L_q} \right| df \quad (17)$$

for canonical scatterers of sufficient length ($L \gtrsim 10\lambda$) and an aperture of sufficient width ($\Theta \gtrsim 10^\circ$). Here, the phase history due to the $q$th canonical scatterer is independent of $\theta$. This result is derived from (11) when expressed in polar coordinates, but a similar result can be obtained for rectangular coordinates using the far-field approximation $r \approx x \cos \theta + y \sin \theta$ and the small-angle approximation $f = \sqrt{f_x^2 + f_y^2} \approx f_y$. Therefore, for rectangularly formatted phase histories, an additional aperture limitation of $\Theta \lesssim 20^\circ$ is required in order to support a small-angle approximation.

Because $L$ is usually not known a priori, it is desirable to modify (17) to account for both distributed and point scatterers with a single parameter $\alpha'$. Adopting the form of (7), a convenient form for (17) is

$$\hat{s}(x_q, y_q) \approx \frac{1}{2\pi} \int_{-\pi}^{\pi} H_B(f - f_q) H_\theta(\theta - \theta_c) \left| \frac{c A_q}{2 f L_q} \right| f |d\theta df \quad (18)$$

![Fig. 5. Wide-angle-approximation relative error for a rectangular window in azimuth and three other windows commonly used in SAR image processing.](image)
are discriminated from their point scatterer variants by tilt model in two dimensions, the distributed canonical scatterers Table II. Aside from the plate, which is symmetric under the point model in (7), and as a result, Table I expands to become SAR. This new model provides an extension of the traditional central to the high-frequency multipeak model for wide-angle now be approximated as due to a canonical point scatterer is of the multipeak model is that, for wide-angle SAR imagery, reasonable elevation angles (e.g., less than 60°) is imaged over multiple wide-angle subapertures, then for the sphere and plate are ambiguous. When the phase history and_png

\[ \alpha' = \alpha - 2 \] and
\[ A' = A - \alpha' \] from (19). Therefore, the SAR image peak amplitude due to a distributed canonical scatterer can be modeled as due to an equivalent canonical point scatterer having an azimuth-independent scaled amplitude function in the spectral domain and a frequency dependence of reduced order.

The fact that image peaks due to any canonical scatterer can now be approximated as due to a canonical point scatterer is central to the high-frequency multipeak model for wide-angle SAR. This new model provides an extension of the traditional point model in (7), and as a result, Table I expands to become Table II. Aside from the plate, which is symmetric under the model in two dimensions, the distributed canonical scatterers are discriminated from their point scatter variants by tilt angles of 0° and 90°, as indicated by a subscript. A benefit of the multipeak model is that, for wide-angle SAR imagery, canonical scatterer tilt angles of \( \tau \approx 0° \) and \( \tau \approx 90° \) can be discriminated without the need for fully polarimetric SAR data.

If fully polarimetric data are available, ambiguity between most scatters can be further resolved using a Krogager decomposition to obtain a ratio of the odd-bounce, even-bounce, and helical scattering [20], [34]. Table III lists the new frequency parameter \( \alpha' \) with the odd- and even-bounce parameters \( \kappa_o \) and \( \kappa_e \), respectively. Using this basis for classification, only the plate and sphere are ambiguous. When the phase history is imaged over multiple wide-angle subapertures, then for reasonable elevation angles (e.g., less than 60°), canonical point scatterers will tend to persist across multiple subaperture images, whereas distributed canonical scatterers do not persist. In this case, a persistence criterion can be used to further resolve ambiguity between distributed and point canonical scatterers, such as the sphere and plate. The papers [30], [31], [35], and [36] and their references contain information on canonical scatterer persistence which, for brevity, is not addressed in this paper.

### IV. Canonical Scatterer Classification Experiments

It is desirable to succinctly present the accuracy of the multipeak model for all canonical scatterer types and at all nonzero orientation angles. In addition, it is helpful to illustrate how the multipeak model in (18) is useful for classification of canonical scatterers in SAR imagery. Consequently, this section illustrates the applicability of the model to canonical scatterer classification while reiterating some of the benefits and limitations of the model which were discussed in the previous section. A simple and efficient least squares classifier based on domain decomposition imaging is preferred to clearly demonstrate the model. Domain decomposition imaging is a method of time–frequency analysis that reduces image resolution to obtain frequency response information on a pixel-by-pixel basis [37], [38].

The classification results in this section were produced using a version of the spectrum parted linked image test (SPLIT) algorithm described in [20]. The SPLIT algorithm produces multiple coarse-resolution subimages from a phase history decomposed into multiple subbands. When a peak persists in all subimages, it is assumed that the peak is dominated by the response due to an ideal canonical scatterer. The corresponding peak intensity samples from all subimages are compared to those predicted by the new multipeak model. For example, in the case of two subimages, the ratio of subimage pixel intensities is related to the subband center frequencies by [20]

\[ \frac{|\tilde{g}_1(x_1, y_1)|^2}{|\tilde{g}_2(x_2, y_2)|^2} \approx \left( \frac{f_{c1}}{f_{c2}} \right)^{\alpha'} \] (20)

using a narrow-band assumption. Note that, when the subimages are reconstructed from annularly shaped phase histories, an additional factor of \( (f_{c1}/f_{c2})^2 \) is required on the right-hand side of (20) to account for the coordinate transformation from a rectangularly shaped phase history. The value of \( \alpha' \) is estimated from the sampled image intensities on the left-hand side and the known center frequencies on the right-hand side. Alternately, when more than two subimages at different subband center frequencies are analyzed, the estimation of \( \alpha' \) becomes a straightforward curve-fitting exercise [20].

Because (20) uses the new multipeak model parameterized by \( \alpha' \), it is instructive to reevaluate the applicability of (18) for use with the SPLIT algorithm. From (12) and assuming that \( \theta_i = 0 \) without loss of generality, the ratio of integrated windowed sinc functions must satisfy the relationship

\[ \frac{\int_{-\pi}^{\pi} H_{E}(\theta) \text{sinc} \left( \frac{\pi}{2} f_1 L \sin(\theta - \theta_0) \right) d\theta}{\int_{-\pi}^{\pi} H_{E}(\theta) \text{sinc} \left( \frac{\pi}{2} f_2 L \sin(\theta - \theta_0) \right) d\theta} = \frac{f_2}{f_1} \] (21)

### Table II

<table>
<thead>
<tr>
<th>Scattering geometry</th>
<th>( \alpha' )</th>
</tr>
</thead>
<tbody>
<tr>
<td>trihedral, dihedral</td>
<td>2</td>
</tr>
<tr>
<td>cylinder, top hat</td>
<td>1</td>
</tr>
<tr>
<td>sphere, plate, edge</td>
<td>0</td>
</tr>
<tr>
<td>cylinder, edge</td>
<td>-1</td>
</tr>
<tr>
<td>edge/cylinder</td>
<td>-2</td>
</tr>
</tbody>
</table>

### Table III

<table>
<thead>
<tr>
<th>Scattering geometry</th>
<th>( \alpha' )</th>
<th>( \kappa_o )</th>
<th>( \kappa_e )</th>
</tr>
</thead>
<tbody>
<tr>
<td>trihedral</td>
<td>2</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>dihedral</td>
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<td>0</td>
<td>1</td>
</tr>
<tr>
<td>cylinder</td>
<td>1</td>
<td>1</td>
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</tr>
<tr>
<td>top hat</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>sphere, plate</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>edge/cylinder</td>
<td>-1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>edge/cylinder</td>
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<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>helical</td>
<td>any</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
where $\Theta > \Theta_{\text{min}}$ and the minimum aperture width $\Theta_{\text{min}}$ can be determined by numerical integration. The numerical results for (21) using a Hanning window are shown in Fig. 6, where $f_1 = 10$ GHz and $f_2 = 9$ GHz. Fig. 6 (right) numerically confirms that (21) is satisfied at a center frequency ratio of 0.9 for $\Theta \gtrsim 10^\circ$ and $L \gtrsim 10\lambda$. Earlier analyses indicate that this relationship holds for varying frequencies and orientation angles as long as the aperture width is restricted to $\Theta \gtrsim 10^\circ$ and the window function in azimuth is tapered.

Using the parameters for ideal canonical scatterers from Table III, a least squares classifier partitions the joint frequency–polarization feature space into regions as shown in Fig. 7. In Fig. 7, $\alpha$ denotes the frequency parameter from (20), and $\kappa_o$ and $\kappa_e$ are the polarization coefficients from the Krogager polarization scattering matrix decomposition. Specific details on how these parameters are extracted and compiled across multiple subapertures are provided in [37].

Simulated phase histories were produced for 15 canonical scatterers listed in Table IV using canonical scattering models from [39]. This data set contains specific examples of the ideal canonical scatters in Table III, to include six different canonical point scatterers and three different distributed canonical scatterers of varying lengths. The lengths were specifically chosen to illustrate the limitations of the high-frequency approximation at X-band. The phase histories were simulated using a circular aperture according to the specifications in Table V. Here, the azimuth angle is referenced counterclockwise from the negative $y$-axis, and the frequency and azimuth are listed as start:increment:end values.

Classification accuracy is dependent upon model accuracy, bandwidth, noise, and clutter. However, in order to isolate the effect of model accuracy on classification accuracy, there was no additive noise or clutter included in the simulation. Assuming an accurate multipeak model, the effects of bandwidth, noise, and clutter on classification accuracy are expected to follow the results already published for the SPLIT algorithm [20].

The images shown in Fig. 8 were produced by integrating the SPLIT-based canonical scatterer classification algorithm with a fast convolution backprojection imaging algorithm as described in [37]. The integrated algorithm employs domain decomposition to produce coarse-resolution subimages for subsequent analysis by the scatterer classifier; then, it interpolates, weights, and aggregates the subimages to form a fine-resolution SAR image. Additional details regarding the integrated algorithm are provided in the Appendix.

Each image in Fig. 8 is truncated to show only the top 45 dB of image intensity. Here, the shape and color of the symbols correspond to the canonical scatterer classes in Fig. 7, and for diagnostic purposes, the symbol size varies with proximity to the ideal canonical scatterer in each class. Thus, a very small symbol indicates an observed feature vector that resides very near a decision boundary between the classes in Fig. 7.

For the case of $\Theta = 10^\circ$ [Fig. 8(a)], a single aperture is used to produce the classification results and images. Here, the classification results are annotated on the image using the symbology from Fig. 7. For this case, all classification results are correct because the multipeak model is accurate when the wide-angle condition is satisfied and all distributed canonical scatterers are greater than ten wavelengths in length. In this
Fig. 8. Classification results for three different subaperture widths $\Theta$. Scatterers (A)–(F) are point scatterers and (G)–(O) are distributed scatterers of varying lengths. For the distributed scatterers, accuracy decreases as $\Theta$ or $L$ decreases. (a) $\Theta = 10^\circ$. (b) $\Theta = 5^\circ$. (c) $\Theta = 2.5^\circ$.

case, the longer distributed canonical scatterers (I), (L), and (O) each produce a group of in-line peaks. The in-line peaks are a manifestation of the Gibbs phenomenon due to the imaging operator and illustrate a characteristic of distributed canonical scatterers from which the multipeak model derives its name.

For the case of $\Theta = 5^\circ$ [Fig. 8(b)], three evenly spaced overlapping subapertures were used to produce the classification results and images. In this case, the classification results are correct for all canonical point scatterers (A)–(F) and for the longer distributed canonical scatterers, as expected. Also, as expected, the classification results are incorrect for the shortest distributed canonical scatterers (G), (J), and (M) because the accuracy of the new multipeak model is degraded for the case of $\Theta = 5^\circ$ and $L = 15\lambda$. These results were predicted by numerical analysis, where Fig. 6 reveals that the model is only accurate when $L \gtrsim 25\lambda$ for the case of $\Theta = 5^\circ$. The two peaks which were correctly classified came from the analysis of the subapertures at $\theta_0 = [-2.5, 2.5]$. It can be seen in Fig. 9 that the model is valid at $\Theta = 5^\circ$ because $\hat{A}$ is essentially independent of frequency for this case. However, the value at $\hat{A}$ is very small when $\Theta = 5^\circ$, which results in low peak intensities. Low peak intensities are a well-known characteristic of long canonical scatterers imaged at off-broadside aspects [11].

For the case of $\Theta = 2.5^\circ$ [Fig. 8(c)], seven evenly spaced overlapping subapertures were used to produce the classification results and images. In this case, the classification results are correct for all canonical point scatterers (A)–(F) and for the longest distributed canonical scatterers, as expected. Also, as expected, the classification results are incorrect for the shorter distributed canonical scatterers (G), (H), (J), (K), (M), and (N) because the accuracy of the new multipeak model has further degraded for this case. The reasons for this are analogous to the case of $\Theta = 5^\circ$, and the results are as expected based on Fig. 6. Evidently, the model is accurate for $L = 60\lambda$ when $\Theta = 2.5^\circ$, as can be seen by examining (I), (L), and (O).

In summary, all classification results, including incorrect classifications, were nicely predicted using the multipeak model. The classification results were correct for the canonical point scatterers (A)–(F) and the longer distributed canonical scatterers (I), (L), and (O) in every case. For the incorrect classifications, the experiment illustrates how model accuracy decreases gradually as $\Theta$ or $L$ decreases. These results indicate that the multipeak model can be used to classify distributed canonical scatterers by comparing the peak intensities of subimages produced by domain decomposition. However, the aperture should be large ($\Theta \gtrsim 10^\circ$), particularly for distributed canonical scatterers of shorter physical length.
Generally, the primary scattering mechanisms on civilian vehicles are cylindrical returns from the top edges of the vehicle and dihedral returns from the sides of the vehicle and the ground, as shown in Fig. 12 [19], [41]. Fig. 13 shows the cylinder and dihedral classifications, respectively, to highlight these primary scattering mechanisms.

Despite the complexity of the target and the simplicity of the canonical scatterer classifier, three dominant scattering mechanisms are identifiable as well classified. First, the cylindrical edges of the cab roof produce a large circular footprint with a radius of about 2.3 m centered at point (−0.3, 0) m. A large circle appears in the image due to the layover effect [3]. Most of the image peaks associated with this mechanism are correctly classified as a cylinder with a tilt angle of 0°. Second, the right angle formed between the rear bumper and tailgate produces a vertical line about 1.5 m long centered at point (2.9, 0) m. The displacement from the tailgate located at x ≈ 2.2 m is due to the layover effect. Most of the image peaks associated with this mechanism are correctly classified as a dihedral with a tilt angle of 0°. Third, the right angles formed between the truck body and the ground plane produce a rectangular footprint about 1.5 m along the y-axis and 4.5 m along the x-axis centered at (0, 0). Many of the image peaks associated with this mechanism are correctly classified as a dihedral with a tilt angle of 0°; however, a preponderance of clutter, particularly from the wheels and wheel wells, compromises the classification results. All three mechanisms are examples of distributed canonical scatterers found in the geometry of the target which are well classified using the multipeak model. These scattering mechanisms are also identifiable for the sedans featured in Fig. 14, where an inner rectangle of horizontal dihedrals is ringed by horizontal cylinders.

We also applied SPLIT to the Gotcha Public Release data [41]. Although Gotcha data only have a 5.6% fractional bandwidth at 640 MHz, they do include four polarization channels over a 360° aperture and are sufficient to observe the capability of the joint frequency–polarization feature space in the SPLIT algorithm. Fig. 15(a) shows an intensity image with an overlay of the attribution using the same feature space as Fig. 7 but a different color scheme. Again, the attribution occurs during image formation, so there is no need to segment the image or select chips from the image before performing canonical scatterer feature extraction. There are two primary areas in the data set: the lower region which is a parking area and the upper region where various calibration targets are arranged. It can be seen that the man-made structures (cars, calibration targets, and storm drains) are nicely identified. There are some attributions in the upper right region that are unknown because the experiment did not control what was in that area.

In Fig. 15(b), a false-color image of a portion of the controlled parking area [the lower right quadrant of Fig. 15(a)] shows the polarization features in the Krogager decomposition as green and red outlines of the parked vehicles, where red denotes single-bounce scattering and green denotes odd-bounce scattering. Although Gotcha data have a limited fractional bandwidth, the roof and hood lines of the vehicles are attributed as primitives with single radii of curvature as expected, and the ground-to-vehicle interaction shows up as double-bounce scatterers (top hats and dihedrals). Some confusion in estimating α is observed and is primarily due to phase errors arising from

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Fig. 10. Facet model of a two-door Toyota Tacoma from 30° elevation. (a) 120° azimuth. (b) 300° azimuth.

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Fig. 11. Annotated SAR image of a simulated Toyota Tacoma pointed in the −x-direction.

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Fig. 12. Dominant backscatter mechanisms for passenger vehicles are (solid lines) single bounce from the top edge and (dotted lines) double bounce from the dihedral formed with the ground plane.

---

Finally, it is instructive to apply the integrated SPLIT-based canonical scatterer classification and domain decomposition imaging algorithm to a more challenging data set. A facet model of the Toyota Tacoma vehicle from the Air Force Research Laboratory civilian vehicle data domes [40] is shown in Fig. 10, and the experimental results are shown in Fig. 11. The truck bed provides a clutter-rich environment to stress the classification algorithm, and an elevation angle of 45° combined with ground plane imaging was chosen so that layover would separate some of the prime scattering mechanisms. Here, again, the shape and color of the symbols correspond to the canonical scatterer classes in Fig. 7, and the symbol size varies with proximity to the ideal canonical scatterer in each class. As a deviation, however, the helical scatterers were colored to improve contrast and to differentiate helical scatterers according to their α parameters (α = −2 for magenta, α = −1 for red, α = 0 for yellow, α = 1 for green, and α = 2 for cyan). This convention is used for Figs. 11, 13, and 14.
Fig. 13. Classification results for the Toyota Tacoma for specific scatterers. (a) Horizontal cylinders only. (b) Horizontal dihedrals only.

Fig. 14. Annotated SAR images of three automobiles, with each pointed in the −x-direction. (a) Sentra. (b) Avalon. (c) Maxima.

an imprecise measurement of the distance between the phase center of the airborne antenna and the scene center [37]. It is important to note that the measured data are not expected to appear the same as observed in Fig. 14 because of the different resolutions and the fact that vehicles in the Gotcha set are not in the center of the image [19]. Nonetheless, the utility and added capability of the joint frequency–polarization feature space are nicely observed. It is important to construct a training data set to establish decision lines in the feature space, which will be the topic of future work.

For these experiments, the multipeak model allowed classification of canonical scatterers to be performed on a pixel-by-pixel basis solely in the time domain. In particular, the multipeak model does not require a user to segment parts of the image for subsequent spectral analysis in order to classify canonical scatterers, as is commonly done. This results in a computationally efficient algorithm that can readily be integrated into a fast domain decomposition imaging algorithm, as explained in [37].

V. CONCLUSION

This paper has presented a new multipeak model to predict the dispersive behavior of subimage peaks due to canonical scatterers. The model was shown to be an improvement over existing peak models because it accounts for common distributed canonical scatterers, including plates at the broadside aspect, as well as dihedrals, cylinders, and edges/wires lying parallel to the imaging plane. For distributed canonical scatterers, the model approximates the amplitude response with a single factor having an inverse frequency dependence. This factor replaces the integration of the main lobe of the sinc-like reflectivity pattern in azimuth as part of the imaging process. The approximation was shown to be valid for wide-angle SAR when the synthetic aperture is greater than 10° in azimuth. Furthermore, the model accuracy was shown to degrade gradually with a decrease in aperture width below 10°. The wide-angle-approximation error was shown to be controllable but is quite high for a rectangular window in azimuth. Therefore, the new multipeak model is recommended for use with tapered windows.

The new multipeak model is useful for canonical scatterer classification by phase history domain decomposition methods. These methods sacrifice some precision in canonical scatterer localization in order to gain efficiency by removing the need for human supervision typically required for image segmentation methods. SAR canonical scatterer classification experiments using simulated data revealed the applicability of the model as well as its benefits and limitations. One particular benefit of the new multipeak model is that, for wide-angle SAR imagery, canonical scatterer tilt angles of near 0° and 90° can be discriminated without the need for fully polarimetric SAR data.
tion technique in [42] while simultaneously extracting feature vectors from the subimages.

Subimages are produced via convolution backprojection of overlapping subdomains of the phase history. The subdomains are defined by weighted, scaled, and shifted subwindows over subbands in frequency and subapertures in azimuth. Halfway overlapping subapertures of equal width are weighted to produce an approximate full-aperture window expressed as 

\[ H_B(\theta - \Theta) \approx \sum_j c_j H_B'(\theta - \Theta_j) \]

where \( c_j < \Theta \) and the subaperture weights are \( c_j \). Likewise, multiple halfband windows in frequency are weighted to produce an approximate fullband window expressed as 

\[ H_B(f - f_c) \approx \sum_i c_i H_B'(f - f_c_i) \]

The halfband is \( B' = B/2 \), and the window weights are \( c_i \). Thus, the subimages are annotated with the subscripts \( i \) and \( j \) as 

\[ \tilde{g}_{ij}(\mathbf{x}', \mathbf{y}') = B \left\{ c_i H_B'(f - f_c_i) c_j H_{2\theta}'(\theta - \Theta_j) \tilde{G}(f, \theta) \right\} \]

where \( (\mathbf{x}', \mathbf{y}') \) represents vectors of the discrete pixel coordinates for the subimages. The pixel spacing of the subimages is adjusted to match the pixel spacing of the original full-domain image by either oversampling, interpolation, or both. In this way, the final image is succinctly expressed as 

\[ \hat{g}(\mathbf{x}, \mathbf{y}) = \sum_j \sum_i I \{ \tilde{g}_{ij}(\mathbf{x}', \mathbf{y}'; \mathbf{x}, \mathbf{y}) \} \]

where \( I \) is an interpolation operator and the \( \tilde{\cdot} \) symbol indicates that the resultant is an approximation of the full-domain image, with pixel spacing given by \( (\mathbf{x}, \mathbf{y}) \). Alternately, a multilook version of the full-domain image is succinctly expressed as 

\[ \tilde{g}(\mathbf{x}, \mathbf{y}) = \sqrt{\sum_j \sum_i I \{ \tilde{g}_{ij}(\mathbf{x}', \mathbf{y}'; \mathbf{x}, \mathbf{y}) \}^2} \]

### ACKNOWLEDGMENT

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### REFERENCES


### APPENDIX

#### INTEGRATED FAST IMAGING AND CANONICAL SCATTERER CLASSIFICATION ALGORITHM

The integrated algorithm, based on that in [37], is summarized here. It performs fast convolution backprojection imaging using a modified single-level version of the domain decompo-


