Linear Systems II
(Discrete-Time Signals & Systems)

Course Introduction

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M. A. Saville, PhD, PE
Lesson Objectives

• Know the breadth and depth of discrete-time linear systems and signals in engineering & science

• References:


  Linear Systems and Signals, B. P. Lathi, Oxford University Press, USA; 2 edition (July 1, 2004)
Discrete-Time Systems & Signals

Analog System → Analog-to-Digital Converter (ADC) → Digital Processing → Digital-to-Analog Converter (DAC) → Analog System

Continuous-time signals & systems

Sampling Theory

Digital signal processing (DSP)

Sample & hold circuit

Continuous-time signals & systems
Scope of DSP

**Space**
- Space photography
- Data compression
- Remote sensing analysis

**Medical**
- Diagnostic imaging
- Electrocardiogram analysis
- Medical data storage/retrieval

**Commercial**
- Image / sound compression
- Movie special effects
- Video conference calling

**Commercial**
- Voice / data compression
- Echo reduction
- Signal multiplexing
- Filtering

**Military**
- Radar, sonar
- Ordnance guidance
- Secure communication

**Industrial**
- Oil and mineral prospecting
- Process monitoring & control
- Nondestructive testing
- CAD and design tools

**Scientific**
- Earthquake recording & analysis
- Data acquisition
- Spectral analysis
- Simulation and modeling

Ref: S. Chapman.
Digital Signal Processing

- Interdisciplinary, relies on
  - Analog electronics
  - Digital electronics
  - Decision theory
  - Analog signal processing
  - Probability and statistics
  - Numerical analysis
  - Communication theory

Ref: S. Chapman.
Example Signals & Systems

- Telecommunications (voice, video, data, messaging) transfer information from one location to another

**Multiplexed signals**
- Separation and recombination done by order of Time, Frequency, Space, Coded, Physical
- Mux > Demux
- Transmission

T-carrier systems simultaneously send voice or data across a telephone provider’s network.

- **T1:**
  - Each channel (timeslot) is sampled at 8,000 times per second using an 8-bit ADC.
  - Each signal represented by 64 Kbps.
  - All voice signals sent at 1.544 Mbps.

- **T3:** Comprises 28 T1 circuits (44.736 Mbps)

**Compressed signals**
- Each sample has 8-bits per sample, but contains some redundant information
- Compression ratio up to 1:32, but with quality trade-offs

**Echo control**
- Echoes from transmission line junction
- Echoes from feedback: (speaker to microphone)
- Cancel expected echo with copy

Ref: S. Chapman.
Example Signals & Systems

- Remote Sensing & Medical Imaging

What is Synthetic Aperture Radar?

- A radar system with unique concept of operation to perform radar imaging mission

http://faculty.washington.edu/kinahan/petct.html
University of Washington


For more on remote sensing applications and career opportunities see course instructor.
Example Signals & Systems

- Image & Audio Processing

Selected Artistic Effects in Microsoft Powerpoint 2010
Sampling (1/2)

- Ideal samples

\[ s[nT] = s(t) \quad n = 0 \ldots M \quad s(t) = a \sin(2\pi ft) \quad t \geq 0 \]

\[ a = 1.28 \text{ volts} \]
\[ f = 1.0 \text{ Hz}, \]
\[ T = 0.10 \text{ sec} \]
Sampling (2/2)

- Actual samples (\(N\)-bits)
  
  \[
  s[nT] = \begin{cases} 
  A_L, & s(nT) \geq A_L \\
  A_l, & A_l \leq s(nT) < A_{l+1} \\
  A_0, & s(nT) < A_0 
  \end{cases}
  \quad l = 0, \ldots, L \\
  L = 2^N - 1
  
  \]

  \[
  A_{l+1} = A_l + \Delta A \quad \Delta A = \frac{A_{\text{max}}}{L} \quad A_0 = -\frac{A_{\text{max}}}{2}
  \]

  \(N = 3, \quad A_{\text{max}} = 3.0\text{volts}\)

  \(N = 5, \quad A_{\text{max}} = 3.0\text{volts}\)
• Digital processing can be simple scaling, shifting, or reflection

\[ s[nT] = b \cdot s(nT), \quad s[nT] = s(nT - \tau), \quad s[nT] = s(-nT), \]

scaling by \( b \), shifting by \( \tau \), reflection about sample number zero

• Digital processing can be logical comprising complex steps

**Low pass filter algorithm**

1. Given signal \( s[nT] \), compute signal’s spectrum

\[ S[qF] = \sum_{n=0}^{N-1} s[nT]e^{-j\frac{2\pi}{N}nqF}, \quad q = 0, \ldots, N - 1 \]

2. For \( qF > F_{\text{max}} \)

\[ S[qF] \leftarrow wS[qF], \quad 0.0 \leq w < 1.0 \]
Digital-to-Analog Conversion

- Most common method is to use zeroth-order hold circuit
  - Hold the value until the next sample
  - Low pass filter to remove undesired high frequencies

Example low-pass filter (LPF)
*Single-stage, Sallen-Key LPF*

LPF implemented in MATLAB using `cheby2.m` & `filter.m`

Ref: Chapman, Ch. 3
Summary

• Discrete-time signals and systems are part of all modern technologies
  – Industrial controls, medical diagnostics & data storage,
  – military sensing,
  – personal to mass communications

• Major components are
  – analog-to-digital converter (ADC),
  – digital processor and algorithms,
  – digital-to-analog converter (DAC)

• Linear Systems II is preparatory for control systems and DSP
  – Course material spans electronics, communications,
  – numerical analysis and algorithms, probability & stats,
  – analog signal processing, decision theory
% Define signal using 'inline'
st=inline('A*sin(2*pi*f*t)','t','f','A'); % continuous-time signal
Fs=100; % Sampling frequency in Hz
t=0:1/Fs:1; % Sample times in sec
f=1; % Signal frequency
sn=st(t(7:10:end),f,2^7/100); % digital signal

% Create graphs
plot(t,st(t,f,2^7/100),'color','k','linewidth',2)
set(gca,'fontsize',12)
xlabel('Time [sec]');
ylabel('Amplitude [volts]');
hold on
h=stem(t(7:10:end),st(t(7:10:end),f,2^7/100),'markerfacecolor','b');
MATLAB Used for Examples (cont.)

% Construct 3-bit quantizer
N=3; Amax=3.0; l=0:1:2^N-1; A0=-Amax/2; dA=Amax/(2^N-1); Al=A0+dA*l;
for n=1:length(Al)-1, I=find(sn>=Al(n)&sn<Al(n+1)); sq(I)=Al(n); end
I=find(sn<Al(1)); sq(I)=Al(1); I=find(sn>Al(end)); sq(I)=Al(end);
set(h,'ydata',sq)
set(gca,'ytick',Al)
set(gca,'ytickLabel',num2str(Al,'%1.2f'))
legend('{\textit{s}}({\textit{t}})', '{\textit{s}}[{{\textit{nT}}}]','{\textit{s}}_3[{{\textit{nT}}}]')

% Construct 5-bit quantizer and graph
N=5; Amax=3.0; l=0:1:2^N-1; A0=-Amax/2; dA=Amax/(2^N-1); Al=A0+dA*l;
for n=1:length(Al)-1, I=find(sn>=Al(n)&sn<Al(n+1)); sq(I)=Al(n); end
I=find(sn<Al(1)); sq(I)=Al(1); I=find(sn>Al(end)); sq(I)=Al(end);
set(h,'ydata',sq)
set(gca,'ytick',Al)
set(gca,'ytickLabel',num2str(Al,'%1.2f'))
legend('{\textit{s}}({\textit{t}})', '{\textit{s}}[{{\textit{nT}}}]','{\textit{s}}_5[{{\textit{nT}}}]')
% Load MATLAB demo file 'handel.mat'
load handel
N=length(y);

% Set delay time to one-ninth of recording
T=ceil(N/9);

% Allocate memory for corrupted signal
s=zeros([N+T,1]);

% Add echo to original signal
s(1:N)=y(1:N); % Original signal component
sound(s,Fs) % Verify original sound quality

s(T+1:T+N)=s(T+1:T+N)+y(1:N); % Add echo starting at time index T
sound(s,Fs) % Verify corrupted sound

sound(s-[zeros(T,1);y],Fs) % Verify sound after echo cancellation
% Set up simulation timing
N=40;
t0=(0:1/8:5-1/8)';
t=(0:1/40:5-1/40)';
for m=0:38, t(5*m+5)=t0(m+2); end; t(end)=t(end-1);

% Create impulse train
y=randn(N,1);

% Create zeroth-order hold signal
x=kron(y,[1;1;1;1;1]);

% Create low pass Chebyshev Type II digital filter
[B,A]=cheby2(4,20,0.15,'low');
% Construct graphics
figure(1)
stem(t0,y,'linewidth',2.5,'color','k','markersize',0.1)
set(gca,'fontsize',14)
xlabel('Time [sec]'); ylabel('Amplitude [volts]');
legend('Impulse train');

figure(2)
plot(t,x,'linewidth',2.5,'color','k')
set(gca,'fontsize',14)
xlabel('Time [sec]'); ylabel('Amplitude [volts]');
legend('Zeroth-order hold');

figure(3)
plot(0:1/40:5-1/40,filter(B,A,x),'linewidth',2.5,'color','k')
set(gca,'fontsize',14)
xlabel('Time [sec]'); ylabel('Amplitude [volts]');
legend('Reconstructed analog signal');