UNCERTAINTY QUANTIFICATION AND
DIMENSION PREDICTION IN FORGING AND
COOLING PROCESSES

A thesis submitted in partial fulfillment
of the requirement for the degree of
Master of Science in Engineering

By

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B.E., University of Mysore, India, 1999

2004
Wright State University
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UNDER MY SUPERVISION BY Badrinarayan K. Belur ENTITLED
Uncertainty Quantification and Dimension Prediction in Forging and Cooling Processes
BE ACCEPTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE
DEGREE OF Master of Science in Engineering

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The objective of this research is to predict the acceptable hot part dimensional tolerance limits immediately after forging. Dimensional limits are dependent on the initial temperature of the part at which the dimensions are measured. The prediction of the geometric dimensions is done by developing a mathematical model utilizing computer based simulations using ABAQUS-DANTE and DEFORM.

The forged part is allowed to cool down to room temperature, during which various metallurgical changes including phase transformation occur in the part. Phase transformation in the part affects the final part dimensions. The final cold part dimensions also depend on various factors such as press accuracy, initial billet temperature, cooling rate, etc., during the forging process. These conditions are the uncertainties during the process. Quantifying these uncertainties in the forging process would aid the designer in developing a more robust forging process design, thus improving productivity and reducing production cost.

Hot part dimension predictor is accomplished by using a surrogate model of the cooling process obtained from Design of Experiments. The accuracy of the dimensional predictor is verified by conducting cooling simulations for the upper and lower dimensional limits predicted. The errors were found to be less than 0.05%.
ACKNOWLEDGEMENTS

Author wishes to express his gratitude and appreciation to Dr. Ramana V. Grandhi for his constant guidance throughout the graduate studies.

This research work is a part of the Project funded by the U.S. Department of Commerce, National Institute of Standards and Technology, Advanced Technology Program, Cooperative Agreement Number 70NANB0H3014 (the Smartsmith™ project). Support of this work is gratefully acknowledged.

Author extends his special thanks to Dr. Rajiv Shivpuri of The Ohio State University, Dr. Tzuy-Shuh Chang of OG Technologies, Inc., Dr. Raghavan Srinivasan and Dr. Ravi Penmetsa, for their ideas and suggestions during this research work. Author also would like to thank Mr. Ben Beaumont, Mr. Jim Hallet and Mr. Michael J. Trathen of Metaldyne Inc., for process inputs. Author would also like to thank Dr. Lynn Ferguson and Dr. Zhichao Li of Deformation Control Technologies, Inc., for providing DANTE the heat treatment software used in this research.
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Chapter 1

Project Relevance

During hot forging, since two to three parts are produced per minute, quality control is one of the major concerns. Quality control techniques can be classified into 100% inspection and random sampling. Since 100% manual inspection is not practical, random sampling is typically used. Random sampling involves selecting a part at a preset interval (time or number of parts) and inspecting it for (i) geometric tolerances (dimensions) and (ii) defects such as cracks, underfill, and folds.

During random sampling, if the sample picked for inspection is found to meet quality requirements, there is still a possibility that defective parts exist in the batch. Thus, the main drawback of random sampling is that some of the defective parts can go unchecked, which can have an adverse affect on the reputation of the company. This project is aimed at developing a system that provides an efficient, 100% inspection with no human intervention.

The system comprises three main divisions (Fig. 1.1): (i) Data Acquisition System (DAS), (ii) Thermo-mechanically Induced Geometric variation estimator (TIG) and (iii) Predictive Process Control Systems (PPCS). DAS comprises the HotEye™ camera from OG Technologies and a system to process the captured image. The camera captures the image of the hot part from which the part dimensions and temperature are to be determined. This data is supplied to TIG. TIG is a real-time online software package
compatible with PPCS and DAS that estimates the dimensional and geometrical relations between the hot and cooled states of a workpiece. The two main objectives of TIG are as follows:

1. Offline design: This involves offline improvement of the process capability and hence reduction in the defect rate by reducing the variation of the output parameters. This is done by designing the best input parameter settings.

2. Real-time improvement: This involves real-time measurement of the output parameters, computation of the geometrical error and the suggestion of changes so as to reduce the defect rate, and movement of the process towards achieving target specifications.

TIG consists of three main modules, the virtual production model (VPM), the geometric comparator (GC), and the process variation estimator (PVE). The VPM models the manufacturing process with both deterministic and stochastic simulations of the forging process and builds a model relating the variation in output parameters with variations in input parameters. The GC compares the part dimensions measured from HotEye™ with acceptable hot part dimensions predicted for the measured part temperature after forging and determines if the part is within target specifications.

The acceptable hot part dimensions are predicted by determining the variations that occur during the forging/cooling process. The process variations are simulated by conducting Design Of Experiments (DOE) within the acceptable range using finite element analysis of the forging and cooling processes. Reliability analysis is conducted on the response fit generated using the DOE points. The data obtained is used to compute the acceptable hot part dimensions after forging for any other input temperature. The
PVE calculates the error between the measured part and the target part and makes suggestions to PPCS on how to move the process toward making parts within target specifications (if the parts are out of tolerance). PPCS analyzes the suggested corrective actions from TIG and implements the corrective measures.

The innovative technologies incorporated in this system are the ability to (i) capture images and determine part dimensions at temperatures close to 1200º C, and (ii) determine the cause of failure during the forging and cooling process. The advantage of this system is that it provides automated 100% inspection and immediate correction to production problems. This reduces production losses that occur under existing random sampling quality control systems when a defective part goes undetected until the next random sample is taken. When a defective part is detected, all the parts produced from the previous random sample until the time of detection must be inspected, further increasing the need for human intervention. Production losses are incurred due to extra inspection cost and time, when additional defective parts are located. Under the proposed system, defective parts are detected as soon as they are produced, and corrective measures are implemented before the next part is produced.

This system is being developed by a consortium of three universities, (i) University of Michigan, (ii) The Ohio State University and (iii) Wright State University, and OG Technologies, Inc., an image processing company based in Ann Arbor, MI. At Wright State University, the effect of manufacturing anomalies in the forging and cooling processes is quantified and hot part dimensions after forging are predicted, which is the focus of this research.
Chapter 2

Introduction

Most of the products in today’s world are designed using sophisticated computer generated models and finite element analysis (FEA). These designs are based on sound physical models but do not take into account the uncertainties that occur during manufacturing. Manufacturing anomalies in the metal forming industry can induce heavy losses (e.g., premature failure of equipment, underfill, folds in part, and double hit) during mass production.

Hot forging, which is extensively used in metal forming, can be subdivided into three main processes, namely, heating, forging, and cooling. Extensive research has been done on forging process design and flash reduction [1]. However, very little work has been done to analyze the cooling process after forging to determine the effect of cooling parameters on the final forged part dimensions. Identifying, quantifying and controlling uncertainties in the forging and cooling processes has never been done.

The first part of this research aims to identify the process uncertainties that effect geometric variations and demonstrates a trade-off method that will aid in the decision process for controlling process parameters to reduce part rejection and production cost. Incorporating these uncertainties into the forging and cooling process design aids in the development of better manufacturing techniques to obtain more reliable parts.
The second part of this research aims to determine the acceptable upper and lower hot part dimensional bounds for the upper and lower dimensional limits on the cold part for any initial temperature. This aids in predicting the acceptance or rejection of the part at an early stage. This information is incorporated into the Thermo-mechanically Induced Geometric variation estimator (TIG). TIG is a real time system developed to determine the acceptance of a forged part based on the hot part dimensions after forging and before cooling. If a defective or out-of-limit part is detected, the system would determine the corrective action for the next part. This plays a significant role in reducing production costs.

In order to study the effect of various parameters on the final part dimensions and determine critical parameters, sensitivity analysis has to be performed. This can be achieved by identifying the parameters that can vary during manufacturing. A few of the parameters are listed below:

i) Initial temperature of the billet before forging

ii) Variations in forging press controls
   a) Stroke length
   b) Thermal effects

iii) Friction in the dies

iv) Part cooling rate

v) Material properties
   a) Thermal expansion
   b) Flow stress

vi) Recrystallization of the material
vii) Billet positioning in the dies

viii) Scales on the billet

ix) Time required to heat the billet

x) Die velocity

xi) Ambient temperature

Many of these are easy to model, but some are complex and difficult to represent mathematically. Modeling scales (metal oxide formation), which are formed when the billet is heated to high temperatures, is a particularly difficult parameter to model.

When steel is heated to above 350° C, an oxide layer forms on the surface. This oxide layer is referred to as scales and is very brittle. It has a tendency to flake away from the forged part, leading to material loss. In addition, any scale particles that are inadvertently left in the die cavities lead to under-filling the die for the next part.

During temperature measurements, the actual temperature value depends on the accuracy of the measuring equipment. Typically, the non-contact measuring instruments depend on the emissivity of the hot metal, material properties, and various environmental factors. This equipment has to be calibrated before use, and to a large extent its accuracy depends on the operator’s skill.

In general, mechanical, pneumatic, or hydraulic presses are used in forging. These presses have moving parts which are heavy and subjected to high temperatures. The high temperatures heat the press components, inducing expansion. In addition, vibrations generated by the ram and other driving components can affect press settings, such as stroke length and die alignment.
The friction between the billet and dies has a direct relationship with the forging loads required to produce the part. Friction also affects the material flow into the dies, thereby affecting the forged part’s final dimensions. Higher friction means increased die wear, which, in turn, leads to defective parts. The quality and quantity of the lubricant used affect the amount of friction between the billet and dies. Improper lubrication also increases die-wear by generating hot spots and stress concentrations in the dies. Lubricant is sprayed on the dies either automatically or manually. Manual spraying can lead to variations in the quantity of lubrication applied, since the quantity and place of application has no fixed reference. In addition, scales, dust, and other particles are often present in the forging plant environment. Some of these particles can make their way into the lubrication tank. As a result, these particles are inadvertently pumped through the system and can temporarily block the nozzle, thus varying the quantity of lubrication on the dies.

When the billets are placed in the dies manually, repeatability is an issue, since a human, however skilled, is prone to errors. Repeatability has a certain degree of error; this would add to the risks that could be coupled with the uncertainties in the manufacturing process. Statistical data could be used to develop models that would be used in the manufacturing design process to reduce the variations due to human errors.

In cases where the billets are placed and moved by robots, accuracy of the robot is considered. Since the robots control system consists of sensors and controllers, the time taken to activate a controller to correct a slack in the system would depend on how fast the control system responds. Computers in the control system perform millions of computations per second. This introduces the possibility that numerical errors could
accumulate with the variations in the process and thus increase the time required to correct the error by the control system.

Weather conditions determine the ambient temperature and humidity on the work floor. The ambient temperature and humidity have a direct correlation with the cooling rate of the part. This can affect the hardness, stresses and geometry of the part.

All of the aforementioned forging and cooling factors play an important role in determining the geometric variations of the forged part. Geometric variations are quantified in terms of part dimensions in this investigation. For the purpose of determining and quantifying uncertainties due to the factors stated above, the computer simulations of the forging and cooling process (case I: metal wheel) are conducted using DEFORM [2], a commercially used package developed by Scientific Forming Technologies Corporation, a Columbus based company.

Finite element simulations of the cooling process are conducted using ABAQUS [3] with DANTE [4] to predict the hot part dimensions (case II: Metaldyne wheel hub). DANTE is developed by Deformation Control Technologies, Inc., and has an extensive material database for heat treatment and cooling process simulations. The inputs for these simulations are part geometry, material properties, kinetic models, cooling rate, and press characteristics. The kinetic models are used to compute the volume fraction of the different transformed phases, which determine the final part dimensions during cooling.

The uncertainties in the process parameters are simulated using Monte Carlo Simulations, and the probability of part occurrence is represented by probability density functions (PDF). During uncertainty quantification, input process parameters can be assumed to follow different distributions such as uniform, normal, Rayleigh, log-normal
and Weibull distributions. In this work normal distribution behavior of random variables is used.
Chapter 3

Background

Conventional design consists of designing the process for various parameters and optimizing these parameters to develop an optimized process. This design procedure is based on the assumption that the design parameters remain constant throughout the process. There are different approaches that designers use to build an acceptable design: (i) experience-based and (ii) deterministic-based.

Experience-based design has been in use for many years. In this process the designer utilizes the experience and knowledge acquired from many years of work in the forging industry to develop a new design. This is a good method to solve problems in existing designs. Since experience-based design requires construction of the dies for every design conceived, it is not a cost-effective method for developing a new design.

Deterministic-based design is a method which was developed to integrate the experience-based design and numerical design processes. This design method involves sophisticated computer-generated models and finite element analysis and is based on sound physical models that are mathematical approximations to represent various process parameters, such as material properties, phase transformation, part geometry, and press characteristics.

Extensive work has been done utilizing the above-mentioned methods toward reduction of material waste [1], die design [5], energy conservation [7], lubricant design
and heat treatment of a forged component to improve the quality of the part during forging. Lubrication systems and die materials are the other aspects of forging that have been addressed during forging process design.

Over the last decade, designers realized that most of the experience-based or computational designs were made reliable by the large factor of safety incorporated into the design. This increased factor of safety meant allowing for extra material in the design. Since complexity, performance requirement, and cost reduction became primary concerns in a competitive market, designers were forced to reduce this factor of safety, while maintaining or improving reliability of the part. This lead to an enormous effort to determine uncertainties in the part or system that cause failure. The high factor of safety provided in the earlier designs was to overcome these uncertainties. Since most of the cost reductions sought were related to aircraft structures, enormous studies related to structural uncertainties were conducted.

As quantifying and simulating uncertainties became more complex, various theories were developed to determine the reliability of the system. Uncertainties can be classified into two distinct types, (i) aleatory and (ii) epistemic uncertainty. Aleatory uncertainty is also known as irreducible or inherent uncertainty. Epistemic uncertainty is a subjective or reducible uncertainty that occurs due to lack of knowledge and data. Formal theories introduced to handle uncertainty include classical probability theory, possibility theory, and evidence theory. All of these deal with the issue of how to determine the degree to which uncertain events are likely to occur.

Probability theory is one of the popular approaches to quantify uncertainties in structural problems. It was developed mostly for aleatory uncertainty and can be well
represented by a probability distribution. The most popular technique in this theory is the Monte Carlo Simulation. It generates random values of aleatory uncertain variables in a target system from a given probability distribution. The model is simulated with these random values to estimate the probability of failure of a component or system. Uncertainties have been integrated into structural engineering design and optimization in the past, but this is the first time that uncertainties are being investigated in the metal forming field. No significant research has been done in determining, quantifying, and incorporating production process uncertainties in the design of the forging process.
Chapter 4

Analysis Procedure

This chapter deals with the theoretical formulation of the inputs used in the cooling process simulation. The process steps followed are represented in figure 4.1. The forging and cooling process simulations are conducted to understand the sensitivity of different input parameters on final part dimensions. The parameters that are considered as inputs to the simulation include kinetic models, material properties, part geometry, initial temperature of the billet, press characteristics, and boundary conditions (Fig. 4.2-4.3). The scientific approach to formulate surrogate models of the forging/cooling process by using Design of Experiments and Response Surface Models is also discussed.

The entire cooling process is divided into a number of time steps for the simulation. In each step, the temperature cooling is computed first. Based on the temperature cooling history in the current time step, the phase transformations are calculated based on the Johnson-Mehl-Avrami equation and Magee equations. Since the latent heat due to the phase transformation influences the temperature distribution, iteration between the heat transfer and phase transformation continues until convergence is achieved. The Newton-Raphson iteration scheme is used in DEFORM-HT to reach convergence. The elasto-plastic material model uses the information from the thermal gradient and phase transformations at the end of the current step to compute the stress/strain and nodal
displacements. The simulation results of the current step are used as input for the next step, and the process continues until the part reaches room temperature.

4.1. Heat transfer analysis

Heat transfer analysis of air cooling a forged part is a transient problem. The temperature distribution is governed by Fourier’s formula (Eq. 4.1):

\[
\rho c \frac{\partial T}{\partial t} = \frac{\partial}{\partial X} \left( K \frac{\partial T}{\partial X} \right) - \sigma_{\tilde{\gamma}} \hat{\varepsilon}^p + L_i \xi_i
\]  

(4.1)

where \( \rho \) is the density, \( \rho_c \) is the heat capacity, \( K \) is the heat conduction coefficient, \( L_i \) is the latent heat due to phase transformation, \( \sigma_{\tilde{\gamma}} \) is stress, \( \hat{\varepsilon}^p \) is the plastic strain rate, and \( \xi_i \) is the volume fraction of the \( i^{th} \) phase transformed. The second and third terms on the right side of the equation represent the heat due to the plastic deformation, and latent heat due to phase transformations, respectively. In the finite element matrix formation, the heat transfer equation is given below (Eq. 4.2):

\[
[K][T] + [C][\dot{T}] = Q(T, f)
\]  

(4.2)

where \( [K] \) is the heat conduction matrix, \( [C] \) is the heat capacity matrix, and \( Q(T, f) \) is the heat load.

The heat load vector includes two main parts: the heat flux between the component’s surface and the surrounding air, and the latent heat due to phase transformation.

\[
Q(T, f) = \int_s h(T - T_c) dS + \int_v \Delta F_i \Delta E_i dV
\]  

(4.3)
where $Q$ is the heat load, $T_e$ is the surface temperature of the component, $T_{th}$ is the environment temperature, $\Delta V_i$ is the volume of the $i^{th}$ phase transformed, $\Delta E_i$ is the latent heat generated by the unit transformation of the $i^{th}$ phase, and $h$ is the overall heat transfer coefficient that combines the influence of convection and radiation on the surface of the component.

The following equation (Eq- 4.4) is used to solve the transient heat transfer equation (Eq- 4.2):

$$T_{t+\Delta t} = T_t + \Delta t[(1-\beta)\dot{T}_t + \beta\dot{T}_{t+\Delta t}]$$

(4.4)

where $\beta$ is a parameter varying between 0 and 1 (usually chosen as 0.75); $t$ denotes the cooling time; $T_t$ and $\dot{T}_t$ are the nodal temperature and the nodal temperature rate at time $t$, respectively; and $T_{t+\Delta t}$ and $\dot{T}_{t+\Delta t}$ are the nodal temperature and the nodal temperature rate at time $t + \Delta t$, respectively. Substitution of equation 4.4 into equation 4.2 results in the following (Eq- 4.5):

$$\left([K] + \frac{[C]}{\beta\Delta t}\right)T_{t+\Delta t} = Q_{t+\Delta t} + \left[C\left(\frac{T_t}{\beta\Delta t} + \left(\frac{1-\beta}{\beta}\right)\dot{T}_t\right)\right]$$

(4.5)

After each time step $\Delta t$, the nodal temperature $T_t$ and nodal temperature rate $\dot{T}_t$ are calculated. During the simulation of the next time step, the only unknown in equation (4.5) is $T_{t+\Delta t}$, which can be solved by the conjugate-gradient method.

### 4.2. Kinetic models

During cooling of forged parts, many metallurgical changes occur, such as phase transformations, microstructure changes, and stress-induced phase transformations.
Kinetic models are the mathematical representation of the physical phase transformations. During phase transformation, the volume of the forged part changes. Since the volume change depends on the type of transformation phase, such as austenite, ferrite, bainite, pearlite and martensite, phase transformation is one of the main inputs for the cooling process simulation. Phase transformations can be classified in many ways. Depending on whether diffusion occurs or not, the phase transformations can be classified as diffusional phase transformation and martensitic phase transformation. The diffusional phase transformation involves a carbon diffusion process which is time dependent. The martensitic phase transformation is diffusionless, and the transformation speed is high. Data obtained from the Time-Temperature-Transformation (TTT) diagram [13] is represented mathematically using the Johnson-Mehl-Avrami (Eq-4.6) equation and Magee’s equation (Eq-4.8) which represent the diffusional and diffusionless phase transformations respectively. These equations are used to determine the volume fraction of the different phases during cooling.

a) Johnson-Mehl-Avrami equation: Transformations from austenite to ferrite, austenite to pearlite, and austenite to bainite are modeled using this equation. Volume fractions for these phases are given as inputs as functions of time and temperature (Eq-4.6) [9].

\[
\xi_p = 1 - \exp(-f_T(T)t^n) \tag{4.6}
\]

\[
f_T(T) = \alpha_1 \left( T - \alpha_2 \right)^{\alpha_4} \left( \frac{\alpha_5 - T}{\alpha_6} \right)^{\alpha_7} \tag{4.7}
\]
where

$\xi_p =$ Volume fraction

$f_T(T) =$ Temperature dependent transformation

$\alpha_1 - \alpha_7 =$ Constants

$T =$ Temperature in Kelvin

$t =$ Time

$n =$ Integer from 1- 4

This equation represents only the diffusional phase transformations. It does not represent the martensitic phase transformation. The martensitic phase transformation is represented by Magee’s equation.

b) Magee’s equation: This equation is used for modeling the phase transformation from austenite to martensite (Eq-4.8) [9].

$$\xi = 1 - \exp(\psi_1 T + \psi_2) \quad (4.8)$$

where

$\xi =$ volume fraction

$\psi_1, \psi_2 =$ constants

$T =$ Temperature in Kelvin

In equations 4.7 and 4.8, $\alpha_1$ to $\alpha_7$, $\psi_1$ and $\psi_2$ are determined from the 50% transformation curve in the TTT diagrams. These equations are also referred to as the kinetic model.

Cooling rate determines the phases in the final part. The volume fractions of the phases computed from these equations during the cooling process simulation determine
the final volume of the component. Equation 4.1 represents all the transformations except the martensitic transformation. Since cooling rate is considered as one of the process parameters, both equations 4.6 and 4.8 have to be considered as inputs for the cooling process simulations.

4.3. Material properties

During cooling, material properties, including thermal and mechanical properties, are different for each transformation phase. During the finite element simulation, these properties are calculated using the Rule of mixtures (Eq-4.9) [9].

\[
\chi = \sum \chi_i \xi_i \tag{4.9}
\]

where \( \chi \) is a specified property, \( \chi_i \) and \( \xi_i \) are the property and volume fraction of the \( i^{th} \) phase in the element respectively.

4.4. Initial temperature

In the case of a hydraulic or pneumatic press, the contact time between the billet and dies can be varied, however for a mechanical press, this time is very short. Since the mechanical press is the most commonly used press in the hot forging industry, it is considered for the forging process simulation. Therefore, a single stage forging process may be assumed as an isothermal process. This would give a uniform initial temperature for the cooling process.
4.5. Response Surface Method

The response surface method is used to develop a surrogate model of the forging and cooling process with part dimensions and forging loads as outputs. This method was initially developed by Box and Wilson [16] and other researchers in the statistical field during the 1950s and found wide application in various engineering fields. In general, a polynomial with variables $x_1, x_2, \ldots, x_k$ is a function, which is a linear aggregate of powers and product of the x’s. For a second order polynomial, the RS model can be expressed as (Eq-4.10)

$$\hat{f}(x, b) = b_0 + \sum_{i=1}^{k} b_i x_i + \sum_{i=1}^{k} b_{ii} x_i^2 + \sum_{i=1, j>1}^{k} b_{ij} x_i x_j$$  (4.10)

where $\hat{f}$ is the system response prediction (forging loads/ part dimensions); $x_i$ and $x_j$ denote design variables which have been normalized such that the upper value is +1; the lower value is -1 and the central value is zero. $k$ is the number of independent variables. The coefficients of this equation are determined from the responses obtained from the DOE. The response surface fit can be a polynomial of various orders such as linear, quadratic, etc. Response surface models are generated for each of the required system responses. These polynomials could represent the forging/cooling process responses and provide analytical models for design.

4.6. Design Of Experiments

To generate data to obtain an accurate response surface model and for efficient performance of experiments (computer simulations to replicate the variations in the
forging and cooling process), a scientific approach has to be considered before the simulations are conducted. The process of statistically planning the experiments is also known as Design Of Experiments (DOE) [15]. This is necessary to obtain a meaningful conclusion from the experimental data. The two basic principles of DOE are replication and randomization. Replication allows the experimenter to obtain an estimate of the experimental error, aiding in the determination of statistical differences in the data. Randomization is the cornerstone underlying the use of statistical methods to allocate the experimental material and the order in which the individual experiments are to be performed. The recommended procedure is

i) Statement of the problem

ii) Choice of variables and values

iii) Selection of response variables

iv) Choice of experimental design

v) Performing the experiments

vi) Data analysis

The number of points at which DOE is required varies according to the order of the response surface model employed. Broadly used DOE schemes include two-level, three-level, and five-level factorial design. To obtain a linear response fit, two-level factorial design is sufficient. But for a more accurate fit, a quadratic polynomial is used, so the design variables should be tried at at least three levels. The total number of experiments is $3^k$ for the three-level factorial design, where $k$ is the number of design variables. For a small number of design variables, three-level factorial design is practical, but as the number of variables increases the cost of simulations become prohibitive.
Data obtained from the DOE are used to build a surrogate model that represents the experiment. After the accuracy of the response surface model is verified, it can be used as a substitute for the finite element simulations in the uncertainty analysis of the forging/cooling process.
Chapter 5

Computational Results

5.1. Axisymmetric metal wheel

This chapter deals with quantifying the forging and cooling process uncertainties and presents a trade-off investigation that can aid a designer in reducing production losses. A simple, axisymmetric metal wheel (Fig. 5.1) with variation in thickness along its cross-section is considered. The forged wheel has a hub thickness of 40 mm, hub diameter of 40 mm and an overall diameter of 160 mm. The 2D model of the wheel is meshed with 1059 finite elements. AISI 4140 is the material used for the hub model [14]. The effects of forging process uncertainties such as heat transfer coefficient, initial billet temperature, variation in stroke length, and friction between the dies and billet on final part dimensions are determined.

The forging process simulation is set up for a mechanical press configuration. The input parameters and their respective values are: die velocity = 300 mm/sec, stroke length = 20 mm, friction factor = 0.3 for fully lubricated condition and 0.7 for unlubricated condition, and initial temperature with lower limit of $800^\circ$ C and upper limit of $1300^\circ$ C. Uncertainty in the stroke length, friction factor and billet temperature are incorporated by varying them in the forging simulations. There is no heat transfer between the dies and the billet is assumed. Part geometry from the forging simulation is then used in the cooling process simulations. The heat transfer coefficient in the cooling simulation is
varied to achieve different cooling rates. The heat transfer coefficient is kept constant throughout the cooling process for section 5.2 to 5.5. In section 5.6, the cooling rate of the part was matched with temperature measurements taken using Thermo Cam, an optical thermal measurement device, of air cooling of a hot steel cylinder to room temperature at The Ohio State University by assigning temperature-dependent heat transfer coefficients in the simulations. The Effect of process variations on the final cold part dimensions is determined. The sensitivity of part dimensions to change in the heat transfer coefficient is the first parameter considered.

5.2. Effect due to Heat Transfer Coefficient

The heat transfer coefficient which determines the cooling rate of the part is varied from 0.01 to 0.09 kW/m² K to determine its effect on the final part dimensions after air cooling. All the other simulation inputs, such as part geometry, material properties, phase transformations models and initial part temperature, are kept constant. Before the effect of heat transfer coefficient on final part dimensions are determined, the temperature difference at four locations during cooling (Fig. 5.2) for a heat transfer coefficient of 0.05 kW/m² K is investigated. The difference in temperature between the points on the surface and those inside the part is only a few degrees (Fig. 5.3), which is due to the ratio of surface area to volume. Thus, the maximum effective stress (von Mises) in the component is much less than the yield stress (Fig. 5.4). The amount of residual stress in the part depends on the part geometry. The residual stress increases as the difference in temperature of the surface and that of the part interior increases.

In this study, three linear final part dimensions of the metal wheel are considered; the outer diameter, hub diameter and hub thickness. These dimensions can be plotted as
either absolute or relative dimensions. When the dimensions are plotted individually, their absolute values are selected. Relative dimensioning represents the wheel’s final dimensions with reference to the initial part dimensions, which can be shown as a percentage change in dimension. The percentage change in dimension is defined as (Eq-5.1).

\[
\text{Relative dimensioning} = \% \text{ Change in dimensions} = \frac{\text{Initial dimension} - \text{Final dimension}}{\text{Initial dimension}} \times 100 \quad (5.1)
\]

The percentage change in dimension for variation in heat transfer coefficient is within 0.05 % (Fig. 5.5) for each of the metal wheel’s three dimensions. This is not a significant variation.

5.3. Effect due to Initial Temperature

The effect of the variation in initial temperature of the billet on part dimensions is determined. For this purpose, part temperatures of 1200° C and 1300° C are used as the initial temperatures for the cooling process, as these are the upper and lower limits used in the forging industry. The upper limit of 1300° C is used because steel starts to melt above this temperature, which is undesirable for forging, and 1200° C is the lower limit since any temperature below that would not provide sufficient flow of material. Using these temperatures as the upper and lower bounds for the initial temperature of the billet before forging, it is seen that the percentage change in part dimension is higher for 1300° C than for 1200° C (Fig. 5.6).

The difference between the percentage change in dimensions for the two initial temperatures is nearly 0.2%. This difference is due to the thermal expansion of steel. In
the case of a metal wheel with a radius of 60 mm, this is not a significant variation, but when the variation is extrapolated for a large part, such as an automotive crankshaft with a length of 600 mm, this would be significant, as the variation would be 1.2 mm over the entire length, leading to part rejection. The percentage change in the dimensions due to the variation in heat transfer coefficients is still within 0.05 %.

5.4. Effect due to stroke length variation

During forging, the press applies pressure on the billet through the dies. There is a possibility that during continuous operation of the press, the stroke could deviate occasionally from the specified tolerance limits of the press (required stroke length is 20 mm). The effect of this stroke length variation is simulated for two cases, the first being a decrease in stroke length by 1 mm (stroke length 19 mm), and the second, an increase in stroke length by 1 mm (stroke length 21 mm). These are the upper and lower bounds for variation in the stroke. This is considered to be a large variation for forging. Average variations typically lie between 0.2 to 0.5 mm. The three linear dimensions, outer diameter, hub diameter, and hub thickness are plotted against various heat transfer coefficients (Fig. 5.7). It is seen that variations in the stroke length causes the hub thickness to increase or decrease significantly, while variations in the stroke length do not cause the other two dimensions, hub diameter and outer diameter, to vary significantly. The plot shows positive and negative variations. This is due to the computation of percentage change in the dimensions. A positive value shows the reduction in dimension of the part due to an increase in stroke length.
5.5. Correlation effect

Hub thickness is the main dimension that is affected by variation in stroke length during forging of the metal wheel, thus the percentage change in thickness is plotted to illustrate the combined effect of initial temperature and stroke length (Fig. 5.8). When two or more uncertainties occur simultaneously, the part can be accepted or rejected depending on the degree of variation in the uncertainties. The part is accepted if the variation in the stroke compensates for the shrinkage in the part. Thus, a slight decrease in stroke length and an initial temperature at the upper limit of 1300º C provides an acceptable part. But a slight decrease in stroke length and an initial temperature at the lower limit of 1200º C produces an oversized part. Increase in stroke length and an upper or lower limit of the initial temperature produces a part that is smaller than the required dimensions, and thus rejected.

5.6. Probability of failure

The above investigation shows that there is no significant change in part dimension due to variations in the cooling rates. Also, previous developments have shown that in a closed die forging, the dimensions perpendicular to the stroke are not affected by the variation in stroke length when a complete die fill is achieved. To determine the probability that a forged part will fail, a three-factorial design of the design variables is used to determine the correlation between the variables. The three-factorial design uses lower, central, and upper limits on the initial temperature (1000-1150-1300º C), friction factor (0.3-0.5-0.7) and variation in the stroke length (19-20-21 mm). A constant cooling rate is assumed for all simulations.
The sensitivities of the forging loads with respect to the design variables are plotted (Fig. 5.9), with their lower limits represented by -1 and upper limits by +1. Of the three design variables, the stroke length is the most sensitive variable. This is due to an exponential increase in forging loads after flash formation. Then, the sensitivities of percentage change in part thickness with respect to the design variables are plotted (Fig. 5.10). Again, the stroke length variation is the most sensitive of the three, followed by initial temperature and friction. When complete die fill is achieved, friction has no direct affect on the final part dimensions, but it has a significant effect on the forging load and die fill.

A three factorial DOE is conducted with the three design variables: initial temperature, friction factor and stroke length. A response surface model is constructed using these design points with the percentage change in part thickness as the response. The $R^2$, for the generated response surface model was found to be greater than 0.99, which is an acceptable model accuracy for the model to be used in the uncertainty analysis. Variations in the design variables are considered to determine the effect of system uncertainties. Normal distribution is considered for the input variables in this case study. Three thousand Monte Carlo simulations (one simulation could be considered as one part manufactured) are conducted on the response models to generate the probability density function (PDF). The PDF aids in visualizing the number of out-of-limit parts.

In the first case (Fig. 5.11a), the initial temperature was set to have a mean value of $1200^\circ$C with a standard deviation of 10, friction factor with a mean of 0.3 and a standard deviation of 0.02, and variation in stroke length with a mean of 19.4 mm and standard deviation of 0.1. Similarly the responses were generated by varying the stroke length to
19.6 (Fig. 5.11b) and 19.8 mm (Fig. 5.11c), and initial temperature to 1200º C. Parts lying in the direction of the left arrow (Fig. 5.11) indicate oversized components, which have to be machined and parts in the direction of right arrow (Fig. 5.11) are undersized parts, which are rejected.

The part acceptance limit for this case is assumed to lie between the required dimension of 22 mm and 22.44 mm, which is 2 % over the required dimension. Parts with dimensions under 22 mm are undersized and above 22.44 mm are oversized. The above investigation showed that if the mean of the stroke length was moved to 19.4 mm all the parts would lie below the upper limit. Because 3,000 simulations is a very small number when considering mass production, the number of simulations was increased to 100,000 to understand the effect on percentage change in thickness. Oversized parts (lower limit) are acceptable since they can be machined to size, but undersized parts (upper limit) are rejected, and thus undesirable.

The tabulated results (Table. 5.1) of the number of parts lying outside the upper and lower limits show that the mean value of stroke length can be moved to 19.4 or 19.6 mm, depending on the acceptable cost. If the mean is set at 19.6 mm, about 5.15 % of the parts are rejected and 0.443 % of parts are machinable. If the mean is moved to 19.4 mm, the number of parts rejected is reduced to 0.01 %, but the number of machinable parts increases to 26.83 %. The decision on placing the mean value depends on two main factors

1) Cost of producing a rejected part: This includes material cost, power consumed, die wear and other administrative costs.
2) Cost of machining extra material from a part: This includes tooling cost, operator
cost and other administrative costs.

If the total cost involved in making the parts that are rejected (condition 1200º C,
19.6 mm) is more than the total cost of machining the extra material from the parts below
the lower limit (condition 1200º C, 19.4 mm), then it would be advisable to move the
mean to 19.4 mm. The trade-off limit is dependent on various factors such as the material
cost, the acceptance limits on the process and other miscellaneous costs, thus making it
part-dependent.
Chapter 6

Hot Part Dimension Prediction

This chapter deals with determining the acceptable hot part dimensions. The Metaldyne front wheel hub (Part no. 4638) is considered as an axisymmetric part and modeled in 2-D (Fig. 6.1a - b). ABAQUS/STANDARD™, in conjunction with DANTE, is used to simulate the air cooling process. DANTE is a user subroutine compatible with ABAQUS/STANDARD and developed by Deformation Control Technology, Inc., a Cleveland based company. It contains the material data such as phase transformation data, specific heat, conductivity, etc. for various metals. ABAQUS/STANDARD is the finite element solver used to simulate the cooling process. Case study I in the previous chapter was conducted using DEFORM; the phase transformation models were acceptable but not accurate. Since the dimensions predicted after cooling depend mainly on the accuracy of the phase transformation models, DANTE is used for this part of the research.

6.1. DANTE subroutine

DANTE is a set of user subroutines that link into the ABAQUS/STANDARD™ ² solver. The DANTE subroutines combine mechanics models, metallurgical transformation models and element diffusion models that couple with ABAQUS/STANDARD's diffusion, thermal and stress/displacement solvers to
accurately calculate the response of steel components to a heat treatment process. The multiphase mechanics model is based on an internal state variable approach to model the mechanical behavior of a solid wherein the volume fraction of each phase is tracked as a function of time. Volume fractions of the phases are denoted by $\Phi$ (Eq 6.1-6.5), with subscripts of $A,F,P,B,$ and $M$ referring to austenite, ferrite, pearlite, bainite, and martensite. Time is represented as $t$, temperature as $T$, Carbon wt. % by $C$. The mechanical properties of each phase are input from the DANTE material datafiles, and the mechanical response of the composite structure as it changes during heat treatment is calculated.

Diffusive mobility functions $\nu_F, \nu_P, \nu_B$ are a function of temperature, while the martensite mobility is a function of carbon.

$$\Phi_A = 1 - \Phi_F - \Phi_P - \Phi_B$$

(6.1)

$$\frac{d\Phi_F}{dt} = \nu_F(T)\Phi_F^{\alpha_F} (1 - \Phi_F)^{\beta_F^{-1}} \Phi_A(\Phi_{F,equil}(T) - \Phi_F), \Phi_F(0) = \Phi_{F0}$$

(6.2)

$$\frac{d\Phi_P}{dt} = \nu_P(T)\Phi_P^{\alpha_P} (1 - \Phi_P)^{\beta_P^{-1}} \Phi_A, \Phi_P(0) = \Phi_{P0}$$

(6.3)

$$\frac{d\Phi_B}{dt} = \nu_B(T)(\Phi_B + \Phi_{EB})^{\alpha_B} (1 - \Phi_B)^{\beta_B^{-1}} \Phi_A(\Phi_{B,static}(T) - \Phi_B), \Phi_B(0) = \Phi_{B0}$$

(6.4)
\[
\frac{d\Phi_M}{dT} = \begin{cases} 
0, & T > M_s \\
\nu_M (C) (\Phi_M + \Phi_{eM})^{\alpha_{\mu} (C)} (1 - \Phi_M)^{\beta_{\mu} (C) - 1} (1 - \Phi_M - \Phi_F - \Phi_P - \Phi_B), & T \leq M_s 
\end{cases} 
\]  
(6.5)

These equations are used by DANTE to compute the volume fractions of various phases during heat transfer simulations and are different from those used by DEFORM in the previous chapter. DEFORM uses the Johnson-Mehl-Avrami equation and Magee’s equation as mentioned in chapter four to compute the volume fractions of the different phases.

6.2. Cooling process computations

The cooling process of the Metaldyne front wheel hub (Part no. 4638) is simulated in two parts: first the temperature change and phase transformations are computed (heat transfer analysis), and then the stresses and dimensions are computed using the temperature data from the first part (stress analysis). Before the computed final part dimensions (Fig 6.1c) are accepted, the volume fractions of the different phases and maximum principle stresses are verified with common trends of industry practice.

Three discrete points (Fig 6.2) in the part are chosen such that location one lies on the surface, location two lies at the center of the part and location three lies at the center of the extruded portion of the wheel hub. Volume fractions of different phases and maximum principle stresses at these three locations are plotted as a function of cooling time (Fig. 6.3 - 6.6).

Since air cooling is a slow cooling process, there is nearly no retained austenite and a very small percentage of quenched martensite at all three locations. Maximum principle stress is higher at the surface than inside the part, following the trend that is expected
during air cooling. To determine the final hot part dimensions, a set of DOE points are considered. The initial dimensions of the part after forging are varied and cooling process simulations are conducted to determine the final cold part dimensions. Cooling rate is kept constant for all DOE. The percentage change in dimensions is computed for initial temperatures of 1200, 1000 and 800º C. With the temperature drop due to heat transfer to the dies and the surroundings, the maximum temperature the part can have after forging is set equal to the acceptable lower temperature limit of the initial billet. Thus, the upper temperature limit for this investigation is set at 1200 °C. The lower temperature limit for this investigation is set at 800º C. This is the average temperature physically measured after forging. DOE is conducted by varying the part dimensions before cooling and the initial temperature of the part before cooling. The results show that the percentage change in cold part dimensions for variation in the hot part dimensions during cooling is nearly constant (Fig. 6.7). Part dimension is not coupled with variation in the other dimensions. For example, the percentage change in cold dimension of D4 does not vary with variations in dimensions D5 to D14.

The percentage change in dimension does change with the initial temperature of the part. It also depends on the starting value of the individual dimensions (Fig. 6.8). Since there is no correlation effect between the dimensions during cooling, mean values of the percentage change in dimensions for each of the individual dimensions at 1200, 1000 and 800º C, respectively, are utilized to obtain a spline fit (cubic). The splined data gives the upper and lower acceptable hot part limits for each of the required dimensions for any given initial temperature. Hot part dimensions obtained from the spline fit are verified computationally by conducting forward simulation for both upper and lower hot part
dimensional limits for an initial temperature of 900º C. The dimensions obtained from the simulations match closely with the required dimensions (Table. 6.1a-b).

This function can be incorporated as a subroutine in the geometric comparator developed by the team at The Ohio State University. The geometric predictor is a C++ code and is shown in the appendix. It consists of two parts: the main program and the spline sub routine. The input variables for this program are: curr_temp = part temperature, curr_dimen = hot part dimensions, and cold_input = cold part dimensions. Dimensional specifications for the part are hard coded into the code. The output variables from this code are: target = hot part limits, cool_dimen = cold part dimensions, and hot_dimen = hot part dimensions. A cubic spline function is used to predict the hot and cold part dimensions.

The outline of the code is as follows: the variables are defined, and the upper and lower dimensions tolerances are assigned. Dimensional response data (percentage change) as a function of temperature are also predefined. This data is then used in the spline subroutine to compute the percentage change of dimensions for the initial part temperature. The final cold/hot part dimensions are computed from the percentage change in dimensions. This code, when compiled and executed independently, would prompt the user to input the part temperature (input) and prints the output as a matrix containing the dimension number and acceptable upper and lower dimensional limits at the input temperature on the screen.
Chapter 7

Conclusions

During production, most parts were found to be within specified limits. Some parts were rejected due to under-fill, folds or double hit, and others were outside the specified dimensional limits. When using more expensive materials such as titanium, aluminum and high grade steel, determining the cause of part rejection can assist in reducing production costs. It also aids in controlling the quality of the part.

Initially, it was believed that the final part dimensions depend only on the cooling process. The results from this research show that the variation in the final part dimensions during air cooling does not depend solely on the cooling rate, but on other factors, such as initial temperature of the billet before forging, friction between the dies and billet, and variations in the forging set up, which, during conventional design, were assumed to be constant throughout the forging process. In reality, most of them vary over a specific range.

Process uncertainties have been identified and computationally verified. The effect of variations in the process parameters on the final dimensions of the part are evaluated in this research work. Also, the cumulative effect of initial temperature and forging press accuracy is considered to demonstrate a coupling effect on the final part dimensions. This helps in understanding that certain combinations of variation in process parameters lead to defective parts. Thus, uncertainties in the process parameters are the root cause of all the defects that leading to part rejection.
Part rejection can be reduced by controlling the tolerance limit on the process parameters. Decreasing the tolerance limit on these parameters would increase production costs. Thus, the decisions on the control limits are very much part dependent. In cases where the cost of controlling the process parameters is high, the trade-off between the number of parts that are acceptable as rejects and parts that need extra machining can be incorporated in the process design to determine the mean values of the process parameters. This leads to improved quality and reduced production cost of the hot forged parts.

When a part is picked at random and quality check is conducted on the part, the part has to be at room temperature before any measurements are taken. Measuring the forged part after it has cooled would mean waiting for 45 to 120 minutes. At the rate of two parts produced a minute, there would be about 90 to 240 parts produced before the quality check on one part is conducted. If the part on which the dimensional check is conducted is found to have a defect, all the parts produced till that point have to be checked, and the cause of the defect must be determined and rectified. If the quality check can be conducted as soon as the part is forged, process parameters that caused the defect can be controlled before the next part is produced.

This investigation has shown that when uniform part temperature is considered before cooling, variation in one dimension during forging does not affect other dimensions after cooling. Each of the dimensions in the part has a different percentage change in value. Since the percentage change in dimension from hot to cold depends mainly on the initial temperature of the part, a mathematical model as a function of temperature is developed to predict the upper and lower dimensional limits on the hot part for the prescribed cold
part limits. Integrating this mathematical model into the geometric comparator would aid in determining the acceptance or rejection of the forged part. Thus, if the geometric comparator determines that a part is not within specified limits TIG would then determine the cause of failure and suggest changes to PPCS. This would reduce the losses due to multiple generations of defective parts.
Appendix

The C Program code to predict the hot part dimensions after forging is as follows.

```c
// Function to determine the Hot part dimensions
#include "fnc.h"
#include<stdlib.h>
#include<stdio.h>
#include <iostream.h>
#include <iomanip.h>
#include <math.h>
#include <string.h>

void main()
{
    double curr_temp, target[2][9];
    double curr_dimen [1][9], cold_input[1][9];

    // Input Temperature
    float ii;
    printf("Enter Part temperature :-");
    scanf("%f", &ii);
    curr_temp=ii;
```
printf("\n The Part temperature is %f C\n",curr_temp);

int flag, i, interval, n;

double HulD[8],HllD[8],Cpd[8],Hpd[8];
double b[11], c[11], d[11], f[11], s, t;

// Set output formatting.

cout.setf(ios::scientific);   // Use scientific notation.
cout.precision(15);           // Show all digits.

// Assign values for n and the data arrays x and f (start indexing at 1
// as in the text).

n = 3;

// Percentage change for D4,D5,D6,D7,D10,D11,D13,D14

// temperature assignment
double x[3]={800,1000,1200};

//Percentage assignment
double tmpy[9][3]=
{0.84, 1.29,1.73},
{0.85, 1.32,1.78 },
{0.87, 1.32,1.74 },
{0.87, 1.4 ,1.95 },
{0.88, 1.37,1.86 },
{0.84, 1.32,1.81 },
{0.84, 1.32,1.81 },
{0.88, 1.35,1.82 },
{0.87, 1.36,1.87 };

cchar varname='D';
int numb[10]={4,5,6,7,10,11,13,14};

// cold part acceptable upper limit

double ulD[9]={78.35,31.9,15.65,16.64,28.5,57,75,32};

// cold part acceptable lowerlimit

double llD[9]={75.85,29.92,14.16,15.14,27.55.5,73.5,30.5};
for(int m=0;m<8;m++)
{
    for (i = 0; i < n; i++)
    {

        f[i] = tmpy[m][i];
    }

// Calculate coefficients defining the cubic spline.

flag = Spline_coeff(n, x, f, b, c, d);

// Check flag, calculate spline values at a few points and
// print the results.

if (flag != 0)
    cout << "Error in data, flag = " << flag << endl;
else
{
    t = curr_temp;
    flag = Spline_value(n, x, f, b, c, d, t, interval, s);

    HulD[m]=ulD[m]+(ulD[m]*s/100); // hot part upper limit
    target[0][m]= HulD[m];
    HllD[m]=llD[m]+(llD[m]*s/100); // hot part lower limit
    target[1][m]=HllD[m];
    printf("%c%d \t%f %f\n",varname, numb[m], target[0][m], target[1][m]);
    Cpd[m]=curr_dimen[m]-(curr_dimen[m]*s/100); // cold part dimensions
    cool_dimen[0][m]=Cpd[m];
    // printf("%c%d \t%f \n", varname, numb[m], cool_dimen[0][m]);
    Hpd[m]=cold_input[m]+(cold_input[m]*s/100); // hot part dimensions
    Hot_dimen[0][m]=Hpd[m];
    // printf("%c%d \t%f \n", varname, numb[m], Hot_dimen[0][m]);
}
    cout << endl;
}
}
The following C code is a subroutine to compute coefficients of the spline equation and evaluate the equation at required value.

```c
// FUNCTION: Functions for setting up and evaluating a cubic interpolatory spline.
#include "fnc.h"

int Spline_coeff(int n, double x[], double f[], double b[], double c[], double d[])
{
    //////////////////////////////////////////////////////////////////////////
    // Calculate coefficients defining a smooth cubic interpolatory spline.
    //////////////////////////////////////////////////////////////////////////
    // Input parameters:
    // n = number of data points.
    // x = vector of values of the independent variable ordered
    //     so that x[i] < x[i+1] for all i.
    // f = vector of values of the dependent variable.
    // Output parameters:
    // b = vector of S'(x[i]) values.
    // c = vector of S''(x[i])/2 values.
    // d = vector of S'''(x[i]+)/6 values (i < n).
    // return_value = 0 normal return;
    //               = -1 input n <= 1;
    //               = -2 x vector is incorrectly ordered.
    //////////////////////////////////////////////////////////////////////////
    // Local variables:
    int i, k;
    double fp1, fpn, h, p;
    const double zero = 0.0, two = 2.0, three = 3.0;

    if (n <= 1) return -1;

    // Calculate coefficients for the tri-diagonal system: store
    // sub-diagonal in b, diagonal in d, difference quotient in c.
    b[0] = x[1] - x[0];
    if (b[0] <= zero) return -2;
    c[0] = (f[1] - f[0]) / b[0];
    if (n == 2)
    {
        b[0] = c[0];
        c[0] = zero;
        d[0] = zero;
    }
    //
    ```
b[1] = b[0];
c[1] = zero;
return 0;
}
d[0] = two * b[0];
for (i = 1; i < n-1; i++)
{
    b[i] = x[i+1] - x[i];
    if (b[i] <= zero) return -2;
    c[i] = (f[i+1] - f[i]) / b[i];
    d[i] = two * (b[i] + b[i-1]);
}
d[n-1] = two * b[n-2];

// Calculate estimates for the end slopes. Use polynomials
// interpolating data nearest the end.
fp1 = c[0] - b[0] * (c[1] - c[0]) / (b[0] + b[1]);
if (n > 3) fp1 = fp1 + b[0] * ((b[0] + b[1]) * (c[2] - c[1])
    / (b[1] + b[2]) - c[1] + c[0]) / (x[3] - x[0]);
if (n > 3) fpn = fpn + b[n-2] * (c[n-2] - c[n-3] - (b[n-3] + b[n-2])
    * (c[n-3] - c[n-4]) / (b[n-3] + b[n-4])) / (x[n-1] - x[n-4]);

// Calculate the right-hand-side and store it in c.
c[n-1] = three * (fpn - c[n-2]);
for (i = n - 2; i > 0; i--)
    c[i] = three * (c[i] - c[i-1]);
c[0] = three * (c[0]-fp1);

// Solve the tridiagonal system.
for (k = 1; k < n; k++)
{
    p = b[k-1] / d[k-1];
    d[k] = d[k] - p * b[k-1];
    c[k] = c[k] - p * c[k-1];
}
c[n-1] = c[n-1] / d[n-1];
for (k = n - 2; k >= 0; k--)
    c[k] = (c[k] - b[k] * c[k+1]) / d[k];

// Calculate the coefficients defining the spline.
for (i = 0; i < n-1; i++)
h = x[i+1] - x[i];
d[i] = (c[i+1] - c[i]) / (three * h);
b[i] = (f[i+1] - f[i]) / h - h * (c[i] + h * d[i]);

b[n-1] = b[n-2] + h * (two * c[n-2] + h * three * d[n-2]);
return 0;

}

int Spline_value(int n, double x[], double f[], double b[], double c[],
double d[], double t, int & interval, double & s)
{
    // Evaluate the spline s at t using coefficients from Spline_coeff.

    // Input parameters:
    //     n, x, f, b, c, d are defined as in Spline_coeff.
    //     t        = point where spline is to be evaluated.
    // Output parameters:
    //     interval = index satisfying x[interval] <= t < x[interval+1]
    //               unless t is outside data interval (see flag).
    //     s        = value of spline at t.
    // return_value =  0  normal return;
    //              = -1  n <= 1;
    //              =  1  t < x[0];
    //              =  2  t > x[n-1].

    // Local variables:
    static int last_interval = 0;
    int flag, j;
    double dt;

    if (n <= 1) return -1;
    flag = 0;

    // Search for correct interval for t.
    if (t < x[0])
    {
        interval = 0;
        flag = 1;
    }
    if (t > x[n-1])
    {
        interval = n - 2;
        flag = 2;
    }
if (flag == 0) {
    if (t >= x[last_interval])
        for (j = last_interval; j < n - 1; j++)
            if (t < x[j + 1])
                interval = j;
        break;
    }
} else
    for (j = last_interval - 1; j >= 0; j--)
        if (t >= x[j])
            interval = j;
    break;
}
last_interval = interval;

// Evaluate cubic polynomial on [x[interval], x[interval+1]].

dt = t - x[interval];
s = f[interval] + dt * (b[interval] + dt * (c[interval] + dt * d[interval]));
return flag;
References


Figure 1.1: Process flow diagram
Figure 4.1: Analysis process flow diagram
Figure 4.2: Input and output parameters for the forging process
Figure 4.3: Input and output parameters for the cooling process

- Geometric distortions
- Final part dimensions
- Surface hardness
Metal wheel

Figure 5.1: Forged metal wheel

Finite element representation of half model

Figure 5.2: Location of temperature data points in the wheel
Figure 5.4: Effective stress vs. time for heat transfer coefficient of 0.05 kW/m² K
Figure 5.3: Temperature drop of data points vs. time for heat transfer coefficient of 0.05 kW/m$^2$ K
Figure 5.5: Dimensional change of metal wheel (outer diameter, hub diameter and hub thickness) with respect to various heat transfer coefficients
Figure 5.6: Dimensional changes of metal wheel (outer diameter, hub diameter and hub thickness) at initial temperature (1200° C and 1300° C)
Figure 5.7: Dimensional changes of metal wheel (outer diameter, hub diameter and hub thickness) for variations in stroke length (+1 mm, -1 mm and no change)
Figure 5.8: Dimensional changes in thickness for cumulative effect of variation in initial temperature (1200° C and 1300° C) and stroke length (+1 mm, -1mm and no change)
Initial temperature (1000 – 1300º C) = (-1,1)

Stroke length (19 – 21 mm) = (-1,1)

Friction factor (0.3 – 0.7) = (-1,1)

Forging Load ($10^6$ N)

Figure 5.9: Sensitivity of the forging load due to variation in design variables (initial temperature, friction factor and stroke length)
Figure 5.10: Sensitivity of change in hub thickness due to variation in design variables (initial temperature, friction factor and stroke length)
Figure 5.11: Probability Density Function (PDF) for variation in mean stroke length
<table>
<thead>
<tr>
<th>Initial Temperature (º C)</th>
<th>Standard Deviation Friction Factor</th>
<th>Standard Deviation Stroke Length (mm)</th>
<th>Standard Deviation</th>
<th>No. of Iterations</th>
<th>Lower Limit (%) Machinable</th>
<th>Upper Limit (%) Rejected</th>
</tr>
</thead>
<tbody>
<tr>
<td>1200</td>
<td>10 0.3 0.02</td>
<td>19.4 0.1</td>
<td>100000</td>
<td></td>
<td>26831(26.83)</td>
<td>18(0.018)</td>
</tr>
<tr>
<td>1200</td>
<td>10 0.3 0.02</td>
<td>19.6 0.1</td>
<td>100000</td>
<td></td>
<td>443(0.443)</td>
<td>5154(5.1540)</td>
</tr>
<tr>
<td>1200</td>
<td>10 0.3 0.02</td>
<td>19.8 0.1</td>
<td>100000</td>
<td></td>
<td>0(0)</td>
<td>64534(64.53)</td>
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<tr>
<td>1250</td>
<td>10 0.3 0.02</td>
<td>19.4 0.1</td>
<td>100000</td>
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<td>19591(19.59)</td>
<td>32(0.032)</td>
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<td>228(0.22)</td>
<td>8,075(8.07)</td>
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<td>100000</td>
<td></td>
<td>0(0)</td>
<td>72619(72.61)</td>
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</tbody>
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Table 5.1: Number of out-of-limit parts based on mean values of initial temperature, friction factor and stroke length
Figure 6.1a: 3-D view of the Hub Front Axle

Figure 6.1b: 2-D section of the Hub Front Axle considered
Figure 6.1c: Dimensional location on the part used for quality control purposes
Figure 6.2: Location at which the volume fraction and stresses are plotted
Figure 6.3: Volume fraction vs. time at location one
Figure 6.4: Volume fraction vs. time at location two
Figure 6.5: Volume fraction vs. time at location three
Figure 6.6: Maximum principle stress vs. time
Figure 6.7: Percentage change in dimension 10 for various initial temperatures
Figure 6.8: Percentage change in dimensions at 1000º C
Table 6.1a: Error between required dimensions (upper limit) and simulated cold part dimensions for initial dimensions predicted by splines

<table>
<thead>
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<th>Dimension</th>
<th>Required</th>
<th>Hot 900 (Upper Limit)</th>
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<td>Actual start</td>
<td>Upper final</td>
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<td>32</td>
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</table>

Table 6.1b: Error between required dimensions (Lower limit) and simulated cold part dimensions for initial dimensions predicted by splines

<table>
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<th>Dimension</th>
<th>Required</th>
<th>Hot 900 (Lower limit)</th>
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</thead>
<tbody>
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