ROLLING MILL OPTIMIZATION USING AN
ACCURATE AND RAPID NEW MODEL FOR
MILL DEFLECTION AND STRIP THICKNESS PROFILE

A dissertation submitted in partial fulfillment of the
requirements for the degree of
Doctor of Philosophy

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ABSTRACT

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Rolling Mill Optimization Using an Accurate and Rapid New Model for Mill Deflection and Strip Thickness Profile.

This work presents improved technology for attaining high-quality rolled metal strip. The new technology is based on an innovative method to model both the static and dynamic characteristics of rolling mill deflection, and it applies equally to both cluster-type and non-cluster-type rolling mill configurations. By effectively combining numerical Finite Element Analysis (FEA) with analytical solid mechanics, the devised approach delivers a rapid, accurate, flexible, high-fidelity model useful for optimizing many important rolling parameters. The associated static deflection model enables computation of the thickness profile and corresponding flatness of the rolled strip. Accurate methods of predicting the strip thickness profile and strip flatness are important in rolling mill design, rolling schedule set-up, control of mill flatness actuators, and optimization of ground roll profiles. The corresponding dynamic deflection model enables solution of the standard eigenvalue problem to determine natural frequencies and modes of vibration. The presented method for solving the roll-stack deflection problem offers several important advantages over traditional methods. In particular, it includes continuity of elastic foundations, non-iterative solution when using pre-determined elastic foundation moduli, continuous third-order displacement fields, simple stress-field determination, the ability to calculate dynamic characteristics, and a comparatively faster solution time. Consistent with the most advanced existing methods, the presented
method accommodates loading conditions that represent roll crowning, roll bending, roll shifting, and roll crossing mechanisms. Validation of the static model is provided by comparing results and solution time with large-scale, commercial finite element simulations. In addition to examples with the common 4-high vertical stand rolling mill, application of the presented method to the most complex of rolling mill configurations is demonstrated with an optimization example involving the 20-high Sendzimir mill.
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1. INTRODUCTION

Despite the advances of numerous metal rolling technologies over the past half-century, intense global competition and the requirements for increasingly thinner, higher quality rolled metal products continue to force metal producers to seek new ways to outperform one another. The need to maximize rolling mill utilization times to achieve profitability, and the significant costs associated with capital upgrades, mean that the application of innovative rolling technologies presents the most attractive near-term solution for many metal producers to improve quality and productivity. At the same time, the companies that build and deliver new rolling mills face increasing pressure and competition to supply their customers with the latest rolling technologies that ensure competitive sustainability. To help address the needs for better rolling technology, presented in this work is an improvement upon the existing process technology for the rolling of high-quality flat metals at high rates of productivity.

1.1 Purpose and Types of Rolling Mills

Metal plate, sheet, or strip (hereafter collectively referred to as “strip”), such as steel, aluminum, or copper is manufactured on hot and cold rolling mills having various configurations of the rolls and with varying numbers of individual rolling stands. A coil of low-carbon steel used for the automobile market is shown in Figure 1a. A hot rolling mill, on which the coil of Figure 1a was manufactured, is shown in Figure 1b. A typical
modern, single-stand cold rolling mill for processing specialty stainless steel is shown in Figure 1c.

Figure 1a – Coil of low-carbon steel used for the automobile market

Figure 1b – One stand of a multiple-stand hot rolling mill

(Photos courtesy of Mittal Steel USA).
Rolling stand configurations for the most common types of hot and cold reduction mills are depicted in Figure 2a. These stand configurations are representative of mills widely employed for large-scale production of hot rolled and cold rolled carbon steel and aluminum sheet. Specialty mills, used to cold roll higher strength and/or thinner gauge materials, such as stainless steels and copper alloys, frequently use “cluster” mill configurations such as those depicted in Figure 2b. Roll clustering is necessary to prevent excessive deflections of the rolls during the conditions of relatively higher loading, as is the case when cold rolling higher strength or thinner gauge metals.

Figure 1c - Modern stainless steel reversing cold mill

(Photo courtesy of Intergrated Industrial Systems, Inc).
2-High  4-High  6-High  4-High Tandem

Figure 2a – Rolling stand configurations for carbon steel mills

2-High  Z-High  6-High  12-High  20-High
(Temper)  (Cluster)

Figure 2b – Rolling stand configurations for high strength, thin gauge steel mills

The purpose of a rolling mill is to successively reduce the thickness of the metal strip and/or impart the desired mechanical and micro-structural properties. Hot rolling mills are used for bulk thickness reduction at elevated temperatures, while cold rolling
mills are employed as secondary rolling operations to achieve more precise dimensional, metallurgical, and mechanical properties. The single-stand type rolling mills are usually operated as “reversing” mills, whereby the strip is successively wound and unwound in coil form as it is repeatedly passed back and forth through the mill. Reversing mills are generally used for smaller scale production of the specialty cold-rolled products. Larger scale production more commonly occurs with tandem-type rolling mills, whereby the strip undergoes a single pass through a train of rolling stands before being wound into coil form. Of all the rolling stand configurations, the 4-high variety is the most widely used – both in single-stand and multi-stand tandem mills. The 2-high mill, which consists of two working rolls only and no other supporting rolls are mainly used for “skin-pass” or temper rolling, the purpose of which is mainly to impart the desired mechanical properties rather than to cause significant reductions in thickness.

1.2 Dimensional Quality Criteria for Rolled Metal Strip

Due to the large magnitudes of applied forces necessary to achieve strip thickness reductions in the rolling process, elastic deflections of the mill housing, rolls, bearings, and other components occur simultaneously with the elastic-plastic deformation of the rolled strip. As a direct consequence of these individual component deflections, two important dimensional quality criteria of the rolled strip arise – 1) thickness profile, and 2) flatness. These dimensional quality criteria are strongly related to the resulting deflection profile of the contact interface between the working rolls and the strip. This deflection profile is typically non-uniform in the direction transverse to rolling by virtue of the geometry of the mill, rolls, and strip. In the absence of corrective measures, the
non-uniform natural deflection at the roll-strip interface causes an uneven strip thickness reduction. Hence, a strip with an initially rectangular cross-sectional thickness profile will typically possess a non-rectangular thickness profile after rolling.

A commonly used metric to measure some aspect of the strip thickness profile is the “crown,” which is defined as the difference in thickness at some point (frequently the center) relative to points near the edges (Figure 3). Although the term “profile” refers to the overall variation in cross-sectional thickness, and “crown” is one metric for profile, the terms “crown” and thickness “profile” are interchanged frequently in industry.

![Figure 3 – Convex strip thickness profile (exaggerated)](image)

Given an initially rectangular strip profile, and without crown-control mechanisms in place, the strip profile that results naturally after rolling is usually convex because the distribution of rolling force is normally greater in the vicinity of the strip edges. The associated strip crown value for a convex profile is positive, since the thickness is greater at the center of the strip width than at the edges. In reality, the influences of various crown-control mechanisms and the thermal and wear effects upon...
the profiles of the rolls combine with the natural strip thickness profile to form various more complex thickness profiles. In this regard, a frequently used, more comprehensive definition of strip crown, \( C(x) \), at any point \( x \) along the strip cross-section, is the difference between the thickness at the center of the strip, \( H(0) \), and the thickness of the corresponding point in question, \( H(x) \), as exemplified in Figure 4 and defined in Equation 1.

\[
C(x) = H(0) - H(x)
\]  

(1)

---

**Figure 4 – Strip crown as a metric of the strip thickness profile**
Frequently, the crown is measured relative to a small distance from both edges and averaged. When the reference distance from either edge (dimension $a$ in Figures 3 and 4) is 25 mm, the crown metric is known as “C25” crown.

The strip crown ratio, $CR(x)$, defined in Equation 2, is the ratio of the strip crown to the centerline thickness and is often expressed in industry as a percentage.

$$CR(x) = \frac{[H(0) - H(x)]}{H(0)}$$  \hspace{1cm} (2)

In most practical cases, the magnitude of C25 crown is relatively small, rarely exceeding five percent of the thickness at the strip centerline. The most important operational issue related to strip crown is the criterion of meeting gauge requirements designated by industry associations such as ASTM International (formerly the American Society for Testing and Materials). For a specific gauge designation, ASTM and similar organizations specify upper and lower tolerances on the rolled metal thickness. Thus, either an excessively convex strip profile with correspondingly large positive strip crown or an excessively concave strip profile with correspondingly large negative strip crown may lead to gauge tolerance violations and rejected product.

A second important dimensional quality criterion closely related to the strip thickness profile is the strip “flatness,” also referred to as “shape”. While crown refers to the transverse non-uniformity of strain in the strip thickness, flatness refers to the
transverse non-uniformity of strain in the strip length, as illustrated in Figure 5. A close relationship exists between crown and flatness because the plastic deformation of the strip is an incompressible process and little or negligible straining of the strip width occurs. Thus, regions across the strip width that undergo relatively greater plastic strain in thickness will undergo correspondingly greater longitudinal plastic strain in the length of the strip. Ideally, if a constant strip crown ratio is maintained, no change in flatness would occur. This principal forms the basis of many flatness control systems.

Figure 5 – Strip flatness as the transverse variation of longitudinal strain

Referring again to Figure 5, the flatness at any point $x$ along the strip width is defined as the magnified longitudinal engineering strain at the same point. It is common in industry to magnify the strain by a factor of $10^5$ to obtain “I-Units.”

$$F(x) = 10^5 \frac{[B'(x) - B]}{B}$$  \hspace{1cm} (3)
An unfortunate confusion arises because the larger the magnitude of flatness according to Equation 3, the less perfect the flatness actually is. A section of strip having perfect flatness, wherein \( F(x) = 0 \) for all \( x \), will lie on a perfectly flat table with continuous contact between the strip and the table at all points. As shown in Figure 6, imperfect flatness can be manifested by excessive longitudinal strain occurring at any region across the strip width. Common flatness defects exhibit distinct patterns of loose regions in the strip, resulting in edge-waves, center-buckles, quarter-buckles, or “herringbone” flatness defects. For an initially rectangular strip thickness profile and perfect initial flatness, the convex strip crown that results naturally after rolling tends to create wavy edges because the larger relative strain in thickness at the strip edges produces correspondingly larger longitudinal strain in the same regions. This phenomenon is commonly referred to in industry as an “over rolling” of the edges. In contrast, an over-rolling condition near the center of the strip width results in center-buckling, an actual example of which on a stainless steel cold mill is shown in Figure 7.

Figure 6 – Common flatness defects with loose regions shaded (viewed from above)
Of the dimension-related quality criteria, strip flatness is the most difficult to control, particularly during the cold rolling of thin materials (less than 0.010 in. or about 0.25 mm). This is because almost no change in width takes place and large changes in the crown ratio and resulting flatness can occur as a result of relatively small absolute changes in the crown. For the opposing reasons, the strip crown is normally controlled during bulk reduction in the early stages of hot rolling, when the material is thicker and significant width strain does occur. Strip flatness, on the other hand, is initially addressed in the latter stages of hot rolling, then in the cold rolling process where its control is paramount. Metal strip lacking suitable flatness poses problems not only in

Figure 7 – Center buckle flatness defect during the rolling of stainless steel
rolling, but in intermediate and downstream operations such as slitting, forming, and stamping of the metal strip.

Since rolled metal strip is used in many applications requiring strict adherence to tolerances, such as in the aerospace, automotive, construction, container, and appliance industries, metals manufacturers must integrate effective profile and flatness control systems into their normal operating procedures. Therefore, the ability to predict and control the thickness profile and corresponding flatness at any stage in the hot or cold rolling process is very important in the manufacturing of high-quality rolled metal products.

1.3 Problem Statement – Accurate and Rapid Model-Based Rolling Optimization

Measures undertaken by metals manufacturers to meet the requirements pertaining to strip profile and strip flatness may include some or all of the following:

1. Use of on-line controls systems that operate rolling mill actuators for the purpose of optimizing the thickness profile and/or flatness during rolling.
2. On-line optimization of pass schedules (thickness reduction schedules) that facilitate the desired profile and flatness.
3. Optimization of suitable ground profiles of the rolls that make possible the desired profile and flatness.
Additional measures undertaken by the manufacturers of the rolling mill equipment and the suppliers of ancillary profile and flatness control mechanisms, respectively, are:

4. Optimum design of the rolling mills to achieve desired profile and flatness.

5. Design of effective and optimal supplemental hardware mechanisms capable of attaining desired strip profile and flatness.

The aforementioned measures – taken by metal producers, rolling mill manufacturers, and suppliers of supplemental profile and flatness control mechanisms – require analytical tools to predict and subsequently control the profile and flatness for a specific mill configuration, control mechanism(s), and rolled material properties. Since the first two items represent on-line activities, their corresponding analytical predictions of strip profile and flatness must not only be accurate but they must be rapid as well. Of the available methods to predict strip profile and corresponding flatness, none satisfy both the accuracy and speed requirements – particularly for cluster-type rolling mills which have more complex geometries and greater numbers of roll-contacting surfaces. Indeed, profile and flatness prediction in cluster-type mills is often addressed either by trial and error, by approximate deflection models for “equivalent” vertical roll-stacks, or by non physics-based pattern recognition models. What is presented herein, therefore, is a profile and flatness model suitable for application to the above measures 1 to 5 with sufficient accuracy and speed for use in on-line systems, and with sufficient flexibility to encompass cluster-type rolling mills. Accordingly, the presented profile and flatness
model must be suitable for incorporation into on-line optimization routines and control system transfer functions, such as the flatness control system for 4-high, single-stand rolling mill depicted in Figure 8.

![Diagram](image.png)

**Figure 8 – Depiction of a strip profile or flatness control system on a 4-high rolling mill**

Although not traditionally considered by methods intended to predict the static deflection of rolling mills for the purpose of calculating strip profile and/or flatness, the ability to predict the dynamic behavior of a rolling mill stand can prevent severe problems in dimensional quality during rolling in addition to avoiding mill hardware damage. By virtue of its construction, the presented profile and flatness model is useful
for dynamic analysis, allowing straightforward computation of natural frequencies and mode shapes of vibration, and the ability to calculate response to harmonic loading, response history analysis, and spectral response evaluation using widely known methods.

1.4 Dissertation Summary

The foundation of the presented work is the development of a rapid and accurate new rolling mill deflection model capable of predicting the strip thickness profile, corresponding strip flatness, and dynamic behavior characteristics for various types of rolling mills. Effectiveness of the model is demonstrated with examples involving optimization of parameters that are important for rolling high-quality metal strip. The presented model addresses the shortcomings of models in existing use; it is suitably fast, accurate, and flexible enough for application with conventional on-line and off-line techniques to control strip profile and flatness on cluster-type and non-cluster-type rolling mills. On-line profile and flatness control techniques involve systems that operate mechanical actuators during rolling, and systems to assign optimal pass schedules immediately prior to rolling. Off-line techniques include optimization of roll grinding profiles, mill design, and design of profile or flatness control mechanisms. In verifying the presented model, examples of its application to on-line profile and flatness control techniques as well as to roll grinding profile optimization are provided. Validation of the model is performed for a single stand 4-High rolling mill by comparing displacement results and solution time with those obtained using large-scale finite element simulations. Additional validation is provided through a Design of Experiments (DOE) study, using large-scale finite element analysis, to evaluate the deflections between contacting rolls.
2. LITERATURE REVIEW

2.1 Background

Rolling research over the past half century to improve the dimensional quality of rolled metal strip has focused primarily in two interrelated areas.

The first general area has dealt with the problem of determining the required rolling force and rolling torque for a specified plastic strain in the thickness of the metal strip. The problem of force and torque determination has been studied extensively since the 1940s [1]. It is an elastic-plastic problem that involves the metal strip, the work rolls, and the interfacial lubricant, as depicted in Figure 9. Major early attempts at solving the plain strain problem were made by Von Karman, Orowan, and Jortner [2-4]. Hitchcock recognized the occurrence of elastic flattening of the work rolls and developed a widely used relationship to estimate the magnitude of a larger effective diameter [5].

Due to the requirements for more practical, real-time calculation of rolling force and torque with less sophisticated solutions, many theorists, including Trinks, Tselikov, Nadai, and Stone applied various simplifying assumptions to the original model developed by Von Karman in 1925 [6-9]. For similar reasons, Orowan’s more general model of 1943 was simplified by Bland & Ford, Underwood, Sims, Ford & Alexander, among others [10-13]. The simplifications and assumptions generally related to contact-
arc form, friction model, yielding criterion, and deformation type (homogeneous or non-homogeneous).

Hypothesizing that the flattened work roll may not remain circular in the arc of contact, and to improve the accuracy of rolling force models for thinner gauges, more recent attempts to solve the plane strain problem were made, for example, by Fleck and Johnson who studied foil rolling [14]. To overcome some of the simplifying assumptions of previous investigators, Wilkund employed the plane-strain slab method and Gratacos used the elastic-plastic finite element method [15-16] to determine required rolling force and torque.

Figure 9 – Plane strain problem of determining rolling force and torque

The second general area of focused rolling research has been to study the problem of the non-uniform deflection of the rolling stand and components (housing, rolls, and strip). This involves the phenomenon that leads to non-uniformities in the strip thickness reduction (with respect to the direction transverse to rolling) and is thus the cause of the strip thickness profile and flatness characteristics introduced previously in Section I. As
depicted in Figure 10, which shows the upper section of a 4-high mill, bending, shear, and surface flattening deflections of the rolls occur simultaneously with the elastic-plastic deformation of the rolled strip. Thus arise the requirements for solving the roll-stack deflection, the distribution of rolling force at the roll-strip interface, and the resulting strip thickness profile. Since thermal expansion of the rolls is often significant, it must be added, accordingly, to any initial roll profiles imparted via mechanical grinding.

Although it is well known that the two general problems of rolling load determination and roll-stack deflection are in fact interrelated and comprise a three-dimensional elastic-plastic problem, the majority of studies have separated the two areas because of the complexity and computation time required for the coupled solution. Recent studies employing boundary element methods and finite element methods for single rolls have been used to solve the three-dimensional problem, but they are yet not practical for modeling mills having multiple rolls or for real-time control system applications [17, 18]. As a result, many prior models to solve the roll stack deflection problem, for the purpose of predicting strip thickness profile and corresponding flatness, arrive at a solution based on a known or assumed distribution of force (often uniform) applied to the work rolls.

In these roll-stack deflection models, the force distribution is simply considered a known input parameter. Models that are able to predict the roll-stack deflection and the distribution of rolling force between the work rolls and strip, but still do not consider the elastic-plastic problem of the strip itself offer an improvement over the earlier models. A
review of the major developments in prior roll-stack deflection models designed to predict strip thickness profile and corresponding flatness is provided next.

![Diagram of roll loading conditions](image)

**Figure 10 – General roll loading conditions combine bending, shear, and flattening deflections**

**2.2 Prior Developments in Mill Deflection Models for Profile and Flatness**

In order to produce a desired strip profile and flatness, metal producers, rolling mill manufacturers, and suppliers of supplemental profile and flatness control mechanisms, require analytical tools to predict and control the profile and flatness according to a specific mill configuration, mechanical control mechanisms in place, and properties of the rolled material. Because of the complexity in modeling rolling mills, particularly those having cluster-type roll configurations, existing methods to predict and control strip profile and flatness have employed analytical methods with various types of simplifying assumptions.
The existing methods to predict strip thickness profile and corresponding flatness of rolled metals can be categorized into five broad methods, listed below and subsequently described:

1. Single-beam on elastic foundation method
2. Influence coefficient / point match method
3. Transport matrix method
4. Pattern recognition / heuristics method
5. Large-scale finite element method

**Single-Beam on Elastic Foundation Method**

The first well-received attempt to calculate roll-stack deflection and predict the strip thickness profile was published by Stone in 1965 [19]. His work studied the effects of work roll bending and back-up roll bending to control strip crown on 4-high rolling mills. In evaluating the effect of work roll bending on strip profile, Stone modeled the work roll as a single Euler-Bernoulli beam on a constant elastic foundation that represented the mutual flattening between the work roll and the back-up roll. Hence, no independent shear or bending deflection of the back-up roll was considered. Flattening between rolls was modeled using Föppl’s plane strain solution of cylinders in lengthwise contact, based on a Hertzian stress distribution, as given by Johnson [20, 21]. When evaluating the effect of back-up roll bending, Stone again ignored shear deformation and only considered the bending stiffness of the back-up roll, while neglecting bending stiffness of the work roll. Despite its many simplifying assumptions, and applicability only to 4-high mills, Stone’s model was used extensively for rough estimates to size
profile and flatness control mechanisms, but was quickly superseded for detailed studies requiring more advanced methods.

**Influence Coefficient / Point Match Method**

After Shohet and Townsend published their influence coefficient method to calculate the effects of roll bending on strip thickness profile, it became the most widely used and most widely adapted method [22, 23]. Models of this type employ a discretized Green’s function (known as an influence coefficient function) to superpose the effects of multiple point loads for the purpose of representing load distributions. “Point matching” is utilized to satisfy equilibrium and compatibility conditions at a finite number of discrete points along the interfaces between the contacting rolls. The method assumes an initial arbitrary force distribution between contacting bodies and uses an iterative procedure to adjust the force distributions to satisfy the point matching. Several improvements and enhancements have been made over the nearly four decades that the method has been used. For instance, Kuhn and Weinstein modified the method to consider the Poisson deflection due to axial bending stresses [24]. Indentation flattening at the interface between the work roll and the strip was considered using Boussinesq’s theory by Kono, then by Tozawa [25-26]. Semi-empirical methods to model the work roll and strip interaction were employed by Nakajima and Matsumoto [27]. Matsubara applied the influence coefficient method to predict the case of mutual contact between upper and lower work rolls during the rolling of foil [28]. Gunawardene used the method to solve for the 20-high cluster mill using an equivalent stack of vertically aligned rolls, and Ogawa extended the method to model 12-high cluster-type rolling mills [29, 30].
Berger enhanced the roll flattening predictions by considering force distribution gradients with respect to roll axial directions [31]. Other investigators, including Pawelski, applied these enhanced models to investigate the effects of roll bending on 4-high and 6-high mills [32]. More recently, Hacquin modified Berger’s basic model to account for a non-elliptical roll-bite stress profile, and changes in the roll flattening behavior near the ends of the rolls based on three-dimensional finite element studies of a single roll [33].

*Transport Matrix Method*

The transport matrix method was used extensively in structural mechanics, but in recent decades was replaced by the more flexible and comprehensive finite element method. An overview of the transport matrix method is provided by Tuma [34]. Its basic concept is to relate a state vector of physical variables, usually shear force, bending moment, slope, and displacement, between two nodes by means of a transport matrix. When several nodes are considered, transport matrices representing interior nodes are successively multiplied together to establish a relationship between two end nodes. Sufficient partial boundary conditions on the end nodes provides the full solution at the end nodes, from which interior nodal solutions can be solved by successive multiplication of respective interior transport matrices. Poplawski was the first to apply the transport matrix method to model the deflection of rolling mills [35]. Guo then applied two-stage and single-stage transport matrix methods to solve a linear spring and beam model of 4-high and 6-high mills, whereby contact between the individual rolls and between the strip and the work rolls was modeled by a finite number of discrete linear springs [36, 37]. A similar linear spring and beam system was applied by Guo to model 20-high cluster mills.
The advantage of the spring and beam model was that it did not require that a known force distribution be applied to the work rolls in lieu of the rolled strip. Instead, because it employed the concept of a linear “strip modulus,” it accommodated perturbations in the distributed force at the roll-strip interface as well as perturbations in the strip thickness strain. These perturbations were deemed acceptable in the vicinity of the nominal operating point.

**Pattern Recognition / Heuristics Method**

Several non physics-based models employing various combinations of fuzzy control algorithms, neural networks, and heuristics methods have been applied, particularly with respect to the modeling of cluster-type rolling mills because of their added complexity. Many of the methods employ training algorithms that receive data automatically from an on-line flatness sensing device. Application of these methods was illustrated by Hattori and Zhu, among others [39-41].

**Large-Scale Finite Element Method**

In the last two decades, boundary element and finite element methods emerged to study the three dimensional problem of the coupled elastic-plastic work roll and strip deflection [17, 18]. Eibe applied the two-dimensional finite element method to study the effect of an inflatable back-up roll upon the strip crown [42]. Chen and Zhou studied strip profile and flatness using a simplified two-dimensional model of a 4-high mill that employed finite elements with variable thickness and moment of inertia to simulate three-dimensional effects [43]. Recently, numerous two-dimensional finite element studies
have emerged to address the plastic flow of material inside the roll-bite. Because of the large number of nodes required, little work, however, has been published recently using commercial finite element methods to solve the two or three-dimensional roll-stack deflection problem for the purpose of computing strip thickness profile and related flatness.

2.3 Unmet Need: An Accurate, Rapid, and Flexible Profile and Flatness Model

Each of the conventional methods to calculate the rolled metal profile and flatness fail to fulfill the need for an accurate, flexible, and rapid model because one of more general shortcomings. It is desirable to obtain a model that is sufficiently accurate and rapid for use with on-line control systems. In addition, a model that comprises the generality to readily consider cluster-type rolling mills in addition to vertical-stack mills is desirable. Accordingly, this work presents a new method to predict profile and flatness that overcomes the principal shortcomings of the conventional models, which are described next.

The first general shortcoming is limited applicability. Because of the inherent complexity, few of the conventional analytical methods to calculate profile and flatness readily encompass cluster-type rolling stand configurations, and the single beam on elastic foundation method is not applicable whatsoever. Of the influence coefficient / point match methods and transport matrix methods that have been devised for use in cluster-type mills, excessively complex models with limited transferability have arisen. For this reason, there is greater prevalence of non-physics based pattern recognition /
heuristics models in predicting and controlling profile and flatness in cluster-type rolling mills.

The second general shortcoming is excessive computation time. The most widely employed method, the influence coefficient / point match method, requires an iterative computational procedure in conjunction with convergence (loop terminating) criteria to obtain a result. Due to the number of iterations and associated computation time, the influence coefficient / point match method is not directly suitable for on-line prediction and control in rolling mills. While the transport matrix method has been used on-line for vertical-stack (non cluster type) rolling mills, it is also not suitably fast enough for mills having relatively large numbers of rolls, such as the 20-roll Sendzimir cluster mill. Large-scale finite element methods require the most computation time of any conventional method. Even for off-line studies, wherein execution time is not critical, the finite element method’s use is questionable because of the convergence issues and lengthy computation time associated with contact-type structural analyses.

The third general shortcoming is insufficient accuracy. The single beam on elastic foundation method is inaccurate in all instances because it neglects shear deformation of the work rolls and considers deflection of the backup rolls (shear, bending, and flattening) as a constant elastic foundation. The influence coefficient / point match method and transport matrix method suffer inaccuracy because the strip profile is predicted only in a piecewise continuous manner, with accuracy conditional upon a relatively large number of closely-spaced nodes. As node count is increased to improve
accuracy, computation time and speed are adversely affected. In addition, since the transport matrix method employs a model of discretely separated nodal springs instead of a continuous elastic foundation that is mathematically integrated, accuracy is sacrificed, particularly in the vicinity of component ends where accuracy is most important. The risk of using discrete springs has been highlighted by Cook [44].

The fourth general shortcoming is the prerequisite of training the profile and flatness prediction or control system with large amounts of data collected from the rolling operation. Since pattern recognition / heuristic models are non-physics based, they exhibit deficiencies in both trend and accuracy in the absence of training with actual data. Such required data may not be available prior to commissioning a strip profile and flatness control system, particularly for newly-started rolling mills.

In addition to the above shortcomings, none of the conventional methods are capable of predicting the dynamic deflection behavior of rolling mills. As mentioned previously, while not traditionally considered by methods that statically model strip profile and flatness, the ability to predict adverse dynamic characteristics of rolling mills can prevent severe problems in dimensional quality in addition to expensive mill equipment damage.

2.4 Literature Review Summary

Rolling research regarding the dimensional quality of the rolled strip has historically been focused in two primary areas; first, with respect to two-dimensional roll-
bite models for predicting the required rolling force and torque, and second, with respect to two and three-dimensional models capable of predicting and controlling strip thickness profile and corresponding flatness. While the two problems are actually interrelated, for convenience and simplicity, steps have been taken to treat them independently. Of the prior profile and flatness methods suitable for on-line application, the linearized spring and beam transport matrix method offers the best compromise because it does not require the input of a known rolling force distribution between the work roll and the strip. The work presented here introduces an improved profile and flatness model that exploits a similar advantage of the transport matrix method while adding the benefits of enhanced accuracy and flexibility.
3. A NEW ROLLING MILL DEFLECTION MODEL

3.1 Introduction

This work presents a novel method to calculate the static deflection of the major components of a rolling mill (housing, rolls, and strip) and predict the resulting thickness profile and corresponding flatness of the rolled metal strip. The method combines the advantages of the conventional finite element method with the advantages of conventional solid mechanics, wherein a compact, accurate, rapid, and flexible method suitable for use in various types of on-line and off-line profile and flatness control techniques is obtained. The method has particular utility in on-line pass-schedule optimization and as a programmed algorithm in computerized profile and flatness control systems that deliver commands to rolling mill profile or flatness actuators. Added utility is realized in off-line applications such as rolling mill design, optimal design of ground roll profiles, or evaluation of profile or flatness control mechanisms.

The generality of the presented deflection model enables consideration of customary rolling mill profile and flatness control mechanisms such as roll mechanical crowning, roll bending, roll shifting, and roll crossing. The incidental effects of roll thermal crowning and roll wearing can be accommodated similarly. Deflection of the rolling mill components is accomplished by creating a linearized global stiffness system, \([K] \mathbf{u} = \mathbf{f}\), that is valid in the vicinity of the expected nominal loading conditions of the
mill stand. Like the discrete spring and beam models of the past developed by other investigators, the presented method has the advantage of not requiring a known force distribution at the interface between the strip and the work rolls. To represent loads applied to the rolling mill, the global system can accommodate any combination of statically equivalent nodal loads, concentrated nodal loads or nodal displacements (Section 3.5). From the nodal displacement vector, \( u \), deflection of the common generator between the work roll and strip, and thus strip profile, can be obtained (Section 3.10). Development of the static linear system is illustrated in the next section, while its derivation is provided in Section 3.3.

### 3.2 Development of the Linear Static Model

As illustrated in Figure 11, Construction of the global stiffness matrix is performed in part by representing individual rolls of a given rolling mill stand as one or more conventional three-dimensional Timoshenko beam finite elements. Construction of the global stiffness matrix is performed further by representing the contact interactions between adjacent rolls as continuous linear elastic foundations, which, in their fundamental form are Winkler (mattress-type) foundations, but which, in their augmented form may be non-Winkler foundations. Such elastic foundations represent linearized load versus center-to-center deflection relationships of cylindrical bodies in lengthwise contact, as may be determined from classical solid mechanics, experimentation, or any other relevant method. Plane-stain analytical methods to determine the elastic foundations representing contact between cylinders may be based upon Hertzian or non-Hertzian pressure distributions, as provided for example by Föppl, Johnson, and
Matsubara [19-21, 28]. Construction of the global stiffness matrix is completed by representing contact interactions between the working rolls and the metal strip by additional continuous linear elastic foundations, which may be derived from any relevant method that provides the sensitivity of the rolling force per unit strip width with respect to strip thickness reduction. As appropriate, any or all of the elastic foundations between the rolls may vary as a function of axial position along the axes of the respective rolls.

Similarly, the elastic foundations between the work roll and the strip may vary as a function along the strip width in the direction transverse to rolling.

To readily accommodate cluster-mill configurations, in which not all rolls are coincident vertically, the method considers angles \( \theta \) for the case when \( \theta \neq \pi/2 \) (Figure 12).
The complete global stiffness matrix is formed by summing the contributions of individual finite element stiffness matrices according to nodal locations in the conventional manner for the well-known finite element method. For elements $i$ of arbitrary beams 1 and 2, each coupled finite element stiffness matrix is formed by combining two three-dimensional Timoshenko beam element matrices with the respective elastic foundation element matrices, as indicated in Equation 4.

$$
[K_{1,2,i}] = [K_{1,2,i}] + [K_{F,1,2,i}]
$$

(4)

Note that matrix $[K_{1,2,i}]$ consists of the Timoshenko beam element contributions for element $i$ of beams 1 and 2 respectively, as shown in Equation 5.

$$
[K_{1,2,i}] = \begin{bmatrix} K_{1,i} & 0 \\ 0 & K_{2,i} \end{bmatrix}
$$

(5)
Similarly, matrix $[K_{F}^{1,2,i}]$ comprises the elastic foundation element contributions for element $i$ between beams 1 and 2, as given in Equation 6.

$$
[K_{F}^{1,2,i}] = \begin{bmatrix}
\int_{0}^{L} k(x)F_{11}(x)dx \\
\int_{0}^{L} k(x)F_{12}(x)dx \\
\int_{0}^{L} k(x)F_{21}(x)dx \\
\int_{0}^{L} k(x)F_{22}(x)dx
\end{bmatrix}
$$

(6)

Terms $F_{pq}$ for $p, q = 1, 2$ in Equation 6 are defined according to Equation 7.

$$
F_{pq} = N_{v_{p}}^{T}N_{v_{q}}\sin^{2}\theta + N_{w_{p}}^{T}N_{w_{q}}\cos^{2}\theta + N_{v_{p}}^{T}N_{w_{q}}\sin\theta\cos\theta + N_{w_{p}}^{T}N_{v_{q}}\sin\theta\cos\theta
$$

(7)

Additional definitions in the foregoing equations are:

- $[K^{n,i}]$ Conventional Timoshenko beam element stiffness matrix for beam $n$, element $i$ (with size 12 by 12 for 6 degrees of freedom per node)
- $N_{v_{n}}$ Vertical displacement shape function sub matrix of Timoshenko beam element shape function matrix $N_{n}$ ($n = 1, 2$)
- $N_{w_{n}}$ Horizontal displacement shape function submatrix of Timoshenko beam element shape function matrix $N_{n}$ ($n = 1, 2$)
- $k(x)$ Foundation modulus between beam elements 1 and 2
- $\theta$ Angle of inclination between beams elements 1 and 2
- $L$ Length of beam elements 1 and 2
If the elastic foundation moduli, \( k(x) \), involve polynomial expressions, the integrals over the element length \( L \) in Equation 6 may be evaluated rapidly by Gauss quadrature.

The element stiffness matrix \( [K_{T^{1,2,i}}] \) given by Equation 4 is a symmetric, positive semi-definite matrix for non-zero \( k(x) \); therefore, the corresponding global stiffness matrix is non-singular and invertible upon removal of rigid-body modes and mechanisms. A schematic of the matrix, \( [K_{T^{1,2,i}}] \), which elastically couples arbitrary beam elements 1 and 2 is shown in Figure 13. It has a size of 24 by 24 when considering all six translational and rotational degrees of freedom per node. The corresponding nodal displacement vector, \( \mathbf{u} \), is defined as:

\[
\mathbf{u} = \begin{bmatrix} u & v & w & \theta_x & \theta_y & \theta_z \end{bmatrix}^T
\]

where \( u \), \( v \), and \( w \), represent translational displacements along the \( x \), \( y \), and \( z \) axes, respectively (as shown previously in Figure 12) and \( \theta_x \), \( \theta_y \), and \( \theta_z \) represent the corresponding rotational displacements. Since axial displacement is normally not considered when computing strip thickness profile, and no axial loading is generally included in the forcing vector, axial degrees of freedom \( u \) may be removed from the global stiffness system, reducing the size of the element stiffness matrix \( [K_{T^{1,2,i}}] \) to 20 by 20. Furthermore, unless it is desirable to include torsion-type elastic foundations, such as may be the case in the corresponding dynamic model discussed in Section 3.12, the torsional degrees of freedom may be removed from the global stiffness system, reducing the element matrix size further to 16 by 16. For non-cluster type rolling mills such as
those with 2-high, 4-high, or 6-high stand configurations, it is not necessary to retain the
degrees of freedom $w$ and $\theta$, and hence the custom element stiffness matrix can be
reduced yet again to a minimum size of 8 by 8.

Since the strip may be assumed to be an elastic foundation only, with no
associated beam properties, the corresponding beam element stiffness matrix for the strip,
$[K^{0,i}]$, may correspond to a zero matrix of the same size. In addition, to avoid duplication
of beam element stiffness contributions for adjacent beams (rolls), zero matrices are
similarly substituted. In Figure 11 shown previously, for example, if element matrix
$[K_T^{1,2,i}]$ contains non-zero beam element contributions $[K^{1,i}]$ and $[K^{2,i}]$ of beams 1 and 2
respectively, then element matrix $[K_T^{0,1,i}]$ will use a zero matrix for beam element
contribution $[K^{1,i}]$. 

Figure 13 – Schematic of the developed finite element stiffness matrix
3.3 Derivation of the Linear Static Model

Derivation of the linear static model to predict the deflection of the rolling mill components and hence strip thickness profile principally involves the derivation of the elastic foundation coupling matrices between the Timoshenko beams that represent the rolls.

Consider first a single beam in the x-y plane of unit width and length L on a fixed Winkler elastic foundation. The additional potential energy due to the elastic foundation, $U_F$, is provided by Cook and cited in Equation 8 with a change in notation such that the foundation modulus per unit length is $k(x)$, and the deflection of the beam against the foundation is $v(x)$ [44].

$$U_F = \frac{1}{2} \int_0^L k(x)v(x)^2 dx$$  \hspace{1cm} (8)

Since the continuous displacement function, $v(x)$, is equal to the product of the vertical displacement shape function matrix, $N_v(x)$, and the y-direction nodal displacement vector, $d_v$, the foundation energy $U_F$ can be written as:

$$U_F = \frac{1}{2} d_v^T \int_0^L k(x)N_v^T N_v dx d_v$$  \hspace{1cm} (9)

The corresponding Winkler foundation element stiffness matrix contribution, $[K_{Fv}]$, is then:
\[
[K_e] = \int_0^L k(x) N_v^T N_v \, dx
\] (10)

Next, instead of a single beam on a fixed elastic foundation, consider the case of an elastic foundation between the axes of two three-dimensional beams 1 and 2, whereby both beams are allowed to move in space, and an angle of inclination, \( \theta \), exists in the y-z plane between the beams. Coordinate geometry \( x, y, z \), and corresponding displacements \( u, v, w \), for the beams are shown in Figure 14. Rotational displacements, \( \theta_x, \theta_y, \theta_z \), are not shown but follow the right-hand-rule convention. Note that \( s_1 \) and \( s_2 \) represent the translational displacements of beams 1 and 2 along the path normal to and directly between the beam center axes.

In this case, the additional potential energy due to the elastic foundation depends on the relative displacement of the axis of one beam with respect to the other, such that \( U_F \) is:

\[
U_F = \frac{1}{2} \int_0^L k(x) [s_1(x) - s_2(x)]^2 \, dx
\] (11)

Based on the coordinate geometry of Figure 14, the terms \( s_n(x) \), for \( n = 1, 2 \), are

\[
s_n(x) = \begin{bmatrix} \sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} v_n(x) \\ w_n(x) \end{bmatrix}
\] (12)

It follows that the term \([s_1(x) - s_2(x)]^2\) in Equation 11 can be written as
\[
\begin{bmatrix}
(s_1(x) - s_2(x))^2 = \\
[(v_1(x) - v_2(x)) (w_1(x) - w_2(x))] \\
\begin{bmatrix}
\sin^2 \theta & \sin \theta \cos \theta \\
\sin \theta \cos \theta & \cos^2 \theta 
\end{bmatrix}
\begin{bmatrix}
v_1(x) - v_2(x) \\
w_1(x) - w_2(x)
\end{bmatrix}
\end{bmatrix}
\]

If we denote the nodal displacement vector of the n^{th} beam as \( \mathbf{d}_n \), and recall that \( v_n(x) = N_{vn} \mathbf{d}_n \) and \( w_n(x) = N_{wn} \mathbf{d}_n \) for \( n = 1, 2 \), Equation 13 can be written in terms of nodal displacements as:

\[
\begin{bmatrix}
(s_1(x) - s_2(x))^2 = \\
\left( (N_1 \mathbf{d}_1 - N_2 \mathbf{d}_2) \right) \left( (N_{v1} \mathbf{d}_1 - N_{v2} \mathbf{d}_2) \right) \\
\begin{bmatrix}
\sin^2 \theta & \sin \theta \cos \theta \\
\sin \theta \cos \theta & \cos^2 \theta 
\end{bmatrix}
\begin{bmatrix}
N_1 \mathbf{d}_1 - N_2 \mathbf{d}_2 \\
N_{v1} \mathbf{d}_1 - N_{v2} \mathbf{d}_2
\end{bmatrix}
\end{bmatrix}
\]

Figure 14 – Coordinate system to define displacement between axes of Beams 1 and 2
It is important to note that for Timoshenko beam elements in general, \( N_{v1} \neq N_{v2} \) and \( N_{w1} \neq N_{w2} \) because the shape function matrices are dependent on the geometric and material properties of beams 1 and 2 respectively, due to the presence of shearing strain terms. The complete shape function matrix \( N \) for a Timoshenko beam is provided by Bazoune and Khulief [45] as follows:

\[
N^T = \begin{bmatrix}
1-\xi & 0 & 0 & 0 \\
0 & \left[1 - 3\xi^2 + 2\xi^3 + (1-\xi)\phi_{x}\right] & 0 & 0 \\
0 & 0 & 1-3\xi^2 + 2\xi^3 + (1-\xi)\phi_{y} & 0 \\
0 & 0 & 0 & 1-\xi \\
0 & 0 & -L\left[\xi - 2\xi^2 + \xi^3 + \frac{1}{2}(\xi - \xi^2)\right] & 0 \\
0 & 0 & 0 & 0 \\
0 & \left(3\xi^2 - 2\xi^3 + \xi\phi_z\right) & 0 & 0 \\
0 & 0 & 0 & \left(3\xi^2 - 2\xi^3 + \xi\phi_z\right) \\
0 & 0 & 0 & 0 \\
0 & L\left[-\xi^2 + \xi^3 - \frac{1}{2}(\xi - \xi^2)\phi_z\right] & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}
\]

where \( \xi = x/L \), and the shear deformation parameters are given by

\[
\phi_{yb} = \frac{1}{(1+\phi_y)}
\]

\[
\phi_y = \frac{12k_y E_b I_z}{A_b G L^2}
\]

\[
\phi_{zb} = \frac{1}{(1+\phi_z)}
\]
\[ \phi_z = \frac{12k_z E_B I_y}{A_b G L^2} \]

\[ k_y = k_z = \frac{(7 + 6v)}{6(1 + v)} \]

The corresponding Winkler foundation element stiffness matrix contribution for the case of two beams with a coupling elastic foundation can be identified upon expansion of Equation 14 and substitution of the result into Equation 11. The operation yields:

\[
U_F = \frac{1}{2} \int_0^L k(x) \left[ s_1(x) - s_2(x) \right]^2 dx
\]

\[
= \frac{1}{2} \mathbf{d}_1^T \int_0^L k(x) \left[ N_{v1}^T N_{v1} \sin^2 \theta + N_{v1}^T N_{w1} \sin \theta \cos \theta + N_{w1}^T N_{v1} \sin \theta \cos \theta + N_{w1}^T N_{w1} \cos^2 \theta \right] dx \mathbf{d}_1
\]

\[
- \frac{1}{2} \mathbf{d}_2^T \int_0^L k(x) \left[ N_{v2}^T N_{v2} \sin^2 \theta + N_{v2}^T N_{w2} \sin \theta \cos \theta + N_{w2}^T N_{v2} \sin \theta \cos \theta + N_{w2}^T N_{w2} \cos^2 \theta \right] dx \mathbf{d}_2
\]

\[
- \frac{1}{2} \mathbf{d}_2^T \int_0^L k(x) \left[ N_{v2}^T N_{v1} \sin^2 \theta + N_{v2}^T N_{w1} \sin \theta \cos \theta + N_{w1}^T N_{v2} \sin \theta \cos \theta + N_{w1}^T N_{w2} \cos^2 \theta \right] dx \mathbf{d}_1
\]

\[
+ \frac{1}{2} \mathbf{d}_2^T \int_0^L k(x) \left[ N_{v2}^T N_{v2} \sin^2 \theta + N_{v2}^T N_{w2} \sin \theta \cos \theta + N_{w2}^T N_{v2} \sin \theta \cos \theta + N_{w2}^T N_{w2} \cos^2 \theta \right] dx \mathbf{d}_2
\]

Hence, a stiffness matrix contribution, \([K_F]_{pq}\), from the coupled elastic foundation terms corresponding to the nodes between the two beams \(p\) and \(q\) can be identified as:

\[
[K_F]_{pq} = (-1)^{pq} \int_0^L k(x) \left[ N_{vp}^T N_{vq} \sin^2 \theta + N_{vp}^T N_{wq} \sin \theta \cos \theta + N_{wp}^T N_{vq} \sin \theta \cos \theta + N_{wp}^T N_{wq} \cos^2 \theta \right] dx
\]
Since the foregoing derivation involves two Timoshenko beam elements with a mutual coupling elastic foundation, the terms in Equation 16 affect a total of 24 degrees of freedom, or 12 degrees of freedom for each beam element. Equation 16 can be written in the form of Equation 6 as a foundation element stiffness matrix for beams 1 and 2. It serves as the coupling matrix between the Timoshenko beam element stiffness matrices of Equation 5. Conventional finite element methods can be used in conjunction with the derived approach to model any structure composed of an arbitrary number of beams coupled elastically along their axes. Such is the case for rolling mills that are used to process flat metal and paper products.

3.4 Elastic Foundation Moduli

The elastic foundation moduli, $k(x)$, introduced previously in Figure 12 and incorporated into Equation 11, represent linearized spring-constants in the relationship between force per unit beam element length and displacement between the beam centers (roll-roll and roll-strip). Unlike in a conventional finite element approach, in which a large number of very small elements may be required to adequately model the contact interface between rolls, the elastic foundations used here represent the “aggregate” displacement-load relationship between the roll axis centers.

Validity of the presented roll-stack deflection model and subsequent method to predict strip thickness profile depends upon the validity of the linearization through the use of linear elastic foundation moduli. At typical magnitudes of distributed load between the adjacent rolls in metal rolling mills, the relationship between displacement
and load is relatively linear. This behavior is shown in Figure 15, which illustrates the center-to-center deflection versus distributed load for a roll of 254 mm diameter in contact with a roll of 508 mm diameter. The inverse of the slope at any given point on each curve is the instantaneous elastic foundation modulus per unit roll length. Figure 15 includes the classic plane-strain analytical solution of Hertz/Föppl [20], the elastic half-space and non-elastic half space solutions of Johnson [21], an analytical solution given by Matsubara [28], and a preliminary conventional FEA solution. Although the FEA solution is preliminary because mesh convergence was not obtained due to the large number of required elements at the contact interface, the results appear to match those of the Hertz/Föppl solution. For this reason, the Hertz/Föppl analytical relationship is used later in Section 3.11 to determine the elastic foundation moduli between rolls during application of the new model to a 4-High rolling mill.

The foundation moduli terms $k(x)$ actually represent the equivalent, series-combined foundation moduli of two individual beam foundations, $k_1(x)$ and $k_2(x)$, representative of either two individual rolls or one work roll and the strip. The equivalent foundation moduli, $k(x)$, can be taken as constants or functions of axial position coordinate $x$. The moduli may be derived from any given state of unit contact force which itself is a function of $x$. In addition, foundation moduli, $k(x)$, can take into account existing roll crowns (combining mechanical grinding, thermal, and wearing effects). Furthermore, the foundation moduli can accommodate roll-crossing and roll-shifting crown-control mechanisms in addition to the effects of applied strip tension stress distributions via their relation with unit rolling force. If desired, for repeated static
deflection calculation with the linearized model, foundation moduli, $k(x)$, may be updated based on unit force distributions from prior calculation to obtain a load-converged solution. The latter option is typically applied in the case of predicted loss of contact between rolls or between rolls and strip, which introduces “hard” non-linearities. It is assumed that the linear model representing the general nonlinear contact problem is valid in the vicinity of an expected, nominal loading condition, at which the foundation moduli between rolls are calculated. Application of the model at other loading conditions undoubtedly reduces the model accuracy.
To determine the foundation modulus between the strip and the work rolls, it is convenient to employ the concept of a linear “strip modulus,” as discussed by Guo [46]. Referring to Figure 16, the strip modulus is the slope of the curve of rolling force per unit strip width versus plastic thickness reduction. The validity of linearization is, of course, related to the linearity of the specific force-reduction relationship for a given material rolled on a particular mill. In examining actual rolling data for an 1880 mm wide mild steel at up to 80 percent thickness reduction, Guo found the use of a linear strip modulus to be satisfactory. In the presented model, we use the same concept of a strip modulus, but replace the discrete nodal springs with a continuous elastic foundation function. The advantage of using this approach over other methods, such as the popular influence coefficient method, is that a known, detailed force distribution at the interface between the work roll and the strip is not required prior to solution—instead, only the average loading condition is needed.

![Figure 16 – Strip foundation modulus](image-url)
During assembly of the global stiffness matrix, the elastic foundations may optionally be converted from Winkler (mattress-type) foundations to non-Winkler foundations by augmenting the global stiffness matrix with modifying stiffness terms, $\Delta K_{ij}$, defined as:

$$\Delta K_{ij} = (-\alpha_{ij})K_{ii}$$  \hspace{1cm} (17)

where individual terms are defined as:

- $\Delta K_{ij}$: change in global stiffness matrix at location corresponding to degrees of freedom $i$ and $j$
- $\alpha_{ij}$: ratio of deflection at degree of freedom $j$ for a unit deflection at degree of freedom $i$
- $K_{ii}$: original global stiffness term for degrees of freedom $i$, $i$

Salimi studied the influence of surface coefficients analogous to terms $\alpha_{ij}$ using commercial finite element analysis and incorporated them into the influence coefficient / point-match method for crown control [47].

### 3.5 Statically Equivalent Loading

Loading of the mill deflection model with forces and/or moments can be applied in two ways:
1. Loads applied at positions that correspond to nodal locations can be directly applied to nodal degrees of freedom.

2. Concentrated or distributed loads not corresponding to nodal locations can be converted into statically equivalent nodal loads [44].

Use of statically equivalent loading is very convenient for use with arbitrary mesh refinement and corresponding element lengths.

3.6 Static Solution of the Global System

Solution of the nodal displacement vector, \( \mathbf{u} \), from the global system, \( [K] \mathbf{u} = \mathbf{f} \), may be accomplished by a variety of methods such as Gaussian elimination or, if reasonable, matrix inversion of \([K]\). The global system, \( [K] \mathbf{u} = \mathbf{f} \), represents a linearization of the general nonlinear contact problem. The general problem is nonlinear because the stiffness matrix \([K]\) is dependent on load vector \( \mathbf{f} \) due to inclusion of foundation moduli \( k(x) \) in global matrix \([K]\), and because the \( k(x) \) terms are derived using load vector \( \mathbf{f} \). It may be convenient and reasonable to use the matrix inversion method of solution in cases where repeated solutions of nodal displacement vector \( \mathbf{u} \) are required for small perturbations in the nominal load vector \( \mathbf{f} \), such as for on-line strip flatness control gain matrix determination, pass-schedule optimization, or other requirements. The use of matrix inversion is reasonable if the all foundation moduli are not highly nonlinear functions of the corresponding load, as was shown earlier in Figure 15 for the case of deflection between the two roll centers at typical magnitudes of rolling force. In this case, matrix inversion needs to be performed once, and repeated solutions are obtained...
using matrix multiplication with the inverse of the global stiffness matrix $[K]$ and the load vector $f$.

3.7 Mesh Convergence

Because of the inclusion of Winkler-type elastic foundation moduli, the accuracy of the presented method depends upon the mesh refinement. For a single beam on an infinitely long elastic foundation, the solution to the deflection of the beam combines sinusoidal and exponential terms [48]. In contrast, beam finite elements involve, at most, cubic polynomial terms for the displacement shape functions. As a consequence, the proposed method delivers an approximate solution. As foundation element length decreases to zero, however, the approximate solution converges to the exact solution of the modeled problem. An example of the mesh convergence behavior of the presented model as applied to a 20-high rolling mill is provided later in Section 3.11.

3.8 Foundation Moduli Convergence

As discussed previously, the general rolling mill deflection problem is nonlinear because the stiffness matrix $[K]$ is dependent on load vector $f$ due to inclusion of foundation moduli $k(x)$ in global matrix $[K]$, and because the $k(x)$ terms are derived using load vector $f$. The type of nonlinearity assumed to exist when employing a non-iterative solution to the linearized problem is known as a “soft” nonlinearity. If any changes in the contacting conditions between the individual rolls, or between the strip and the work rolls occur, including the opening or closing of gaps, then the linearity is known as a “hard” nonlinearity. Hard nonlinearities are detected in the solution by the presence of a tensile
unit force distribution within any elastic foundation, and provide very important information regarding potential adverse operating conditions of a rolling mill. For example, if the incoming strip profile into the rolling stand is excessively convex and the total applied rolling force is insufficient to close all potential gaps in the roll-bite, then the presented model should predict tensile foundation force distributions in the corresponding regions. It should be noted, however, that the nodal displacement solution vector will be inaccurate in such cases, as it is not physically possible to realize a tensile unit force distribution between individual rolls or between the strip and the work rolls. Although, due in particular to the linear behavior of the roll-stack deflection, the presented model may be employed in a non-iterative manner, when large variations in the unit contact force or hard nonlinearities are present, the solution can be repeated using the most current elastic foundation unit force to update the corresponding elastic foundation moduli. Such iteration may be performed until assigned convergence criteria are met.

3.9 Displacement, Strain, and Stress Fields

Superposition of the static global solution, \([K] \mathbf{u} = \mathbf{f}\), and the solution of contact between cylinders enables calculation of the three-dimensional displacement field, strain field, and stress field along the beam axes and at common generator locations. Solution throughout the rest of the beams (rolls and strip) may or may not be straightforward depending on the particular contact problem formulation chosen.
3.10 Force Distribution, Common Generator Displacement, and Thickness Profile

Following solution of the displacement field, the unit rolling force distribution between adjacent rolls or between the work rolls and strip is calculated by multiplying the corresponding foundation modulus with the displacement function for respective custom finite elements. The common generator displacement between the work rolls and the strip determines the strip thickness profile (and crown) of the rolled strip. To calculate the strip crown the vertical position, \( y(x) \), of the common generator surface between the strip and the work roll at the desired axial location \( x \) must be calculated. The common generator vertical position, \( y(x) \), between arbitrary beams 1 and 2 can be obtained using Equation 18:

\[
y(x) = y_{ij}(x) + \left( \frac{D_1(x) - k_1(x) I(x)}{k(x)} \right) \frac{1}{2} \sin \theta
\]  

(18)

In Equation 18, \( y_{ij}(x) \) is the solved vertical position for node \( j \) at the axial coordinate \( x \) of beam 1 (in this case the strip). \( D_1(x) \) is the original diameter of beam 1, which in this case refers to the strip thickness. \( k(x) \) is the equivalent foundation modulus beam between 1 and the adjacent beam (upper or lower work roll), and \( k_1(x) \) is the foundation stiffness contribution of beam 1, which here is the strip modulus. As shown earlier in Figure 12, \( \theta \) is the angle of inclination between adjacent beams (\( \pi/2 \) for the upper work roll or \( -\pi/2 \) for the lower work roll). The term \( I(x) \) in Equation 18 represents the total interference between the adjacent beams, as determined from the original nodal coordinates, the diameter profiles, and the nodal displacements. Equation 18 can be derived from a free body diagram of the nodes connecting beam 1 and the adjacent beam,
noting that the ratio of the foundation displacement magnitudes is the inverse of the ratio of the foundation stiffness moduli:

\[
\frac{\Delta_1(x)}{\Delta(x)} = \frac{k(x)}{k_1(x)}
\]  (19)

In Equation 19, \( \Delta_1(x) \) is the magnitude of the displacement of the foundation modulus \( k_1(x) \) between the surface and the axis of beam 1. \( \Delta(x) \) is the magnitude of the displacement of the equivalent foundation modulus \( k(x) \) between the axis of beam 1 and the axis of the adjacent beam.

As discussed earlier, end-users of rolled metal strip are usually interested in determining the C25 strip crown value. To predict this, one only needs to evaluate the common generator (strip profile) at points that correspond to six locations between the strip and work roll (for a full non-symmetric model). Therefore, it is only necessary to multiply the six corresponding row vectors from \([K]^{-1}\) with the specified load vector \(f\). Thus, use of \([K]^{-1}\) makes the C25 crown determination very rapid.
3.11 Static Solution Examples

To demonstrate the applicability of the new model for mill displacement and strip thickness profile, examples involving the vertical stand 4-High rolling mill and the 20-High Sendzimir cluster-type mill shown in Figure 17 are provided.

**Application of New Model to 4-High Mill**

Application of the new model to simulate the deflection in a 4-High rolling mill of Figure 17 is provided here. Later, in Section 4, the predicted strip profile from the new model is compared with that obtained using a large-scale commercial Finite Element Analysis package. Partial symmetry of the 4-High roll configuration was exploited, leading to an upper half model of the mill. Dimensions of the 4-High mill components are shown in Table 1. They include 1270 mm work roll and back-up roll lengths, a 254 mm work roll diameter, a 508 mm back-up roll diameter, and a 508 mm strip width. The
entry strip thickness is 25.4 mm and the exit thickness at the strip center is 21.077 mm, yielding 17.02% thickness reduction.

Table 1 – Geometry parameters for 4-High mill

<table>
<thead>
<tr>
<th>Geometry Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strip entry thickness, ( H ) (mm)</td>
<td>25.400</td>
</tr>
<tr>
<td>Strip exit thickness, ( h ) (mm)</td>
<td>21.077</td>
</tr>
<tr>
<td>Strip width, ( w ) (mm)</td>
<td>508.00</td>
</tr>
<tr>
<td>Work roll diameter, ( D_w ) (mm)</td>
<td>254.00</td>
</tr>
<tr>
<td>Work roll length, ( L_w ) (mm)</td>
<td>1270.00</td>
</tr>
<tr>
<td>Backup roll diameter, ( D_b ) (mm)</td>
<td>508.00</td>
</tr>
<tr>
<td>Backup roll length, ( L_b ) (mm)</td>
<td>1270.00</td>
</tr>
</tbody>
</table>

Table 2 – Parameters for application of new model to 4-High mill

<table>
<thead>
<tr>
<th>Model Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strip foundation modulus, ( \beta ) (N/mm(^2))</td>
<td>13790</td>
</tr>
<tr>
<td>Strip foundation modulus modification length, ( d ) (mm)</td>
<td>25.00</td>
</tr>
<tr>
<td>Strip foundation modulus end nodes ratio, ( f_1 )</td>
<td>0.50</td>
</tr>
<tr>
<td>Backup roll boundary condition type on end nodes</td>
<td>pinned</td>
</tr>
<tr>
<td>Work roll boundary condition type on end nodes</td>
<td>free</td>
</tr>
<tr>
<td>Strip lower edge vertical disp. boundary condition (mm)</td>
<td>6.35</td>
</tr>
<tr>
<td>Backup roll elastic modulus, ( E_b ) (GPa)</td>
<td>206.84</td>
</tr>
<tr>
<td>Backup roll Poisson ratio, ( v_b )</td>
<td>0.30</td>
</tr>
<tr>
<td>Work roll elastic modulus, ( E_w ) (GPa)</td>
<td>206.84</td>
</tr>
<tr>
<td>Work roll Poisson ratio, ( v_w )</td>
<td>0.30</td>
</tr>
<tr>
<td>Number of Timoshenko beam elements</td>
<td>48</td>
</tr>
</tbody>
</table>

Table 2 indicates model parameters for simulating the 4-High mill. A total of 48 Timoshenko beam elements with associated coupling foundations are used to model the upper half of the mill. The strip foundation modulus, \( k(x) \), is assigned a constant value, \( \beta = 13790 \text{ N/mm}^2 \), over the strip width, \( w \), except for a modification to decrease the foundation stiffness beginning at points \( x = \pm x_0 \), corresponding to a distance \( d \) from either strip edge. The magnitude of \( x_0 \) is therefore \( (w/2-d) \).
The specific assignment of $k(x)$ is indicated in Equations 20a and 20b. The parameter $f_1$ in Equation 20b represents the fraction of the nominal strip foundation modulus $\beta$ remaining at the strip edges $x = \pm w/2$. A value of 0.5 is intuitively used for $f_1$, since the nodes at $x = \pm w/2$ share equally an interior foundation modulus $\beta$, and no external foundation. These equations provide a parabolic decrease in $k(x)$ from a value of $\beta$ at $x = \pm x_0$ to a value of $0.5\beta$ at $x = \pm w/2$. It is widely accepted in rolling operations that a significant decrease in the thickness of the rolled strip occurs within approximately 25 mm from the edges of the strip. This “edge-drop” phenomenon is a principal reason why the standard C25 crown metric evolved. Accordingly, a value of 25 mm is intuitively assigned for the parameter $d$ in Equation 20b.

$$k(x) = \beta, \quad |x| < x_0 \quad (20a)$$

$$k(x) = \beta \left[ \frac{(f_1 - 1)}{d^2} |x - x_0|^2 + 1 \right], \quad |x| \geq x_0 \quad (20b)$$

To simulate the thickness reduction in the half-model, a uniform vertical displacement boundary condition of 6.35 mm is applied to the lower nodes of the strip upper half section. Rigid body motion is prevented by assigning pinned boundary conditions to the end nodes of the upper back-up roll. Although, in this case, a displacement boundary condition is applied to simulate the strip thickness reduction, a force boundary condition, or any combination of boundary conditions, may also be used.
For example, the nodes on the lower edge of the strip might be fixed, while a vertical force boundary condition is applied to the end nodes of the backup roll.

Figures 18a and 18b show the vertical displacement, \( v(x) \), in the \( y \)-direction and the horizontal displacement, \( w(x) \), in the \( z \)-direction, as a function of the \( x \)-direction along the axes of rolls and the strip. As expected, the horizontal displacements are exactly zero since all components of the 4-High rolling stand are coincident vertically and no horizontal loads are imposed. The vertical displacement magnitudes are reduced, respectively, from the strip to the work roll and from the work roll to the back-up roll due to compression of the coupling elastic foundations between the components. Furthermore, greater vertical displacement occurs in the vicinity of the strip, with the maximum vertical displacement occurring at the strip with center.

Figure 18c illustrates the resulting contact force distribution at the interface between the strip and the upper work roll, and between the upper work roll and the back-up roll. Figure 18d shows the thickness profile of the upper half of the strip relative to the semi-thickness at the strip edge. By Equation 1, the strip crown \( C(x) \) corresponding to C25 locations (\( x = \pm 229 \) mm) is 1.118 mm, since the semi-thickness is 0.559 mm greater at the strip center than at the C25 edge locations. Table 3 summarizes the results for the 4-High mill simulation. The model predicts that for a 17.02 % reduction in thickness at the strip center, the thickness at a distance of 25 mm from either edge of the strip is 1.118 mm less than the center thickness (19.959 mm versus 21.077 mm). Hence, the C25 strip crown is 1.118 mm, or 5.304 % of the center thickness.
Table 3 – Results summary for application of new model to 4-High mill

<table>
<thead>
<tr>
<th>New Model Results Summary</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strip center thickness, (h) (mm)</td>
<td>21.077</td>
</tr>
<tr>
<td>Strip C25 thickness, (h_{c25}) (mm)</td>
<td>19.959</td>
</tr>
<tr>
<td>Strip crown, C25 (mm)</td>
<td>1.118</td>
</tr>
<tr>
<td>Strip crown, C25 (%)</td>
<td>5.304</td>
</tr>
<tr>
<td>Total force, (F) (MN)</td>
<td>33.949</td>
</tr>
</tbody>
</table>

Figure 18 – 48 element model results for 4-high rolling mill

a) vertical displacement distribution, \(v(x)\)
b) horizontal displacement distribution, \(w(x)\)
c) contact force distribution
d) thickness relative to edge for upper half of strip
Since this simulation does not include the effect of any crown control devices, Figures 18c and 18d illustrate typical deflection and load characteristics that occur in a 4-High rolling mill. It can be seen that the increase in the contact force distribution in the vicinity of the strip edges leads to greater corresponding thickness reduction in those areas, and hence the evolution of the positive strip crown. Solution time for the 4-High mill half-model using MATLAB, including matrix construction, nodal solution by Gaussian elimination, and post-processing to determine contact force and displacement fields, was approximately one second. While the rolling mill geometry in this example has three planes of symmetry, only two planes are used. Application of the remaining symmetry plane bisects the rolls and strip, decreasing by problem size by half and further reducing the solution time.

**Application of New Model to 20-High Sendzimir Mill**

In order to demonstrate the flexibility of the new model to accommodate complex rolling mill configurations, we now apply the model to the upper section of the 20-High Sendzimir mill depicted earlier in Figure 17. This type of mill has proven very difficult to model with the conventional methods discussed in Section 2. One feature of the Sendzimir mill that poses problems for most conventional strip profile models is that each of the eight outermost rolls are not actually solid rolls. Instead, they are comprised of segmented bearings mounted on common shafts. This arrangement serves two purposes. First, it promotes greater mill rigidity by providing additional support to the surrounding mill housing at the intermediate locations along each shaft. Second, it accommodates bending of the shafts via application of normal loads between the
bearings. This allows some control over the strip profile and flatness. Although the segmented backing rolls have discontinuous contact with the second intermediate rolls, this circumstance is readily accommodated in the new model by using a zero foundation modulus over the corresponding regions. In addition, the discontinuous section moduli of the beams are treated by assigning nodes at locations where the roll diameters and/or material properties change abruptly, as is customary in conventional finite element modeling.

The dimensions of the strip and rolls for the 20-High mill example are shown in Table 4. The entry and exit thickness at the center of the strip are 0.9779 mm and 0.9063 mm respectively, giving 7.32% reduction. The strip width is 508 mm and the length of all rolls is 1270 mm. As shown in Table 4, the roll diameters increase progressively from the work roll to the backing bearing rolls. Each backing bearing roll has six equally spaced bearings of 292.10 mm diameter, mounted on common solid shafts of 127.0 mm diameter. Parameters assigned to the 20-High mill model are shown in Table 5. The upper half of the 20-High mill is modeled using 252 Timoshenko beam elements and associated coupling foundations. A constant strip foundation modulus, $\beta = 52472$ N/mm$^2$, was assigned over the strip width, w, except for the same modification to decrease the modulus in the vicinity of the strip edges described previously in Equations 20a and 20b.
Table 4 – Geometry parameters for 20-High mill

<table>
<thead>
<tr>
<th>Geometry Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strip entry thickness, $H$ (mm)</td>
<td>0.9779</td>
</tr>
<tr>
<td>Strip exit thickness, $h$ (mm)</td>
<td>0.9063</td>
</tr>
<tr>
<td>Strip width, $w$ (mm)</td>
<td>508.00</td>
</tr>
<tr>
<td>Work roll diameter, $D_w$ (mm)</td>
<td>50.800</td>
</tr>
<tr>
<td>Work roll length, $L_w$ (mm)</td>
<td>1270.0</td>
</tr>
<tr>
<td>1st intermediate roll diameter, $D_f$ (mm)</td>
<td>101.60</td>
</tr>
<tr>
<td>1st intermediate roll length, $L_f$ (mm)</td>
<td>1270.0</td>
</tr>
<tr>
<td>2nd intermediate roll diameter, $D_s$ (mm)</td>
<td>172.72</td>
</tr>
<tr>
<td>2nd intermediate roll length, $L_s$ (mm)</td>
<td>1270.0</td>
</tr>
<tr>
<td>Backing bearing outer diameter, $D_{bb}$ (mm)</td>
<td>292.10</td>
</tr>
<tr>
<td>Backing bearing shaft length, $L_{bb}$ (mm)</td>
<td>1270.0</td>
</tr>
<tr>
<td>Backing shaft outer diameter, $D_{bs}$ (mm)</td>
<td>127.00</td>
</tr>
<tr>
<td>No. backing bearings per shaft, $N_{bb}$</td>
<td>6</td>
</tr>
</tbody>
</table>

Table 5 – Parameters for application of new model to 20-High mill

<table>
<thead>
<tr>
<th>Model Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strip foundation modulus, $\beta$ (N/mm²)</td>
<td>52472</td>
</tr>
<tr>
<td>Strip foundation modulus modification length, $d$ (mm)</td>
<td>25.00</td>
</tr>
<tr>
<td>Strip foundation modulus end nodes ratio, $f_i$</td>
<td>0.50</td>
</tr>
<tr>
<td>Backing bearing boundary condition type on all nodes</td>
<td>pinned</td>
</tr>
<tr>
<td>Other roll boundary condition type on end nodes</td>
<td>free</td>
</tr>
<tr>
<td>Strip lower edge vertical disp. boundary condition (mm)</td>
<td>0.5588</td>
</tr>
<tr>
<td>Elastic modulus of all rolls, $E_r$ (GPa)</td>
<td>206.84</td>
</tr>
<tr>
<td>Poisson ratio of all rolls, $\nu_r$</td>
<td>0.30</td>
</tr>
<tr>
<td>Number of Timoshenko beam elements</td>
<td>252</td>
</tr>
</tbody>
</table>

To simulate the thickness reduction, a uniform vertical displacement boundary condition of 0.5588 mm was applied to the lower nodes of the strip upper half section. Rigid body motion was prevented by assigning pinned boundary conditions to each node between the individual bearings of the upper backing rolls. Discrete ground springs may be added at these locations to simulate some elastic compliance in the mill housing. No translation boundary conditions were imposed on any other roll.
Figures 19a and 19b show the vertical displacement, $v(x)$, in the $y$-direction and the horizontal displacement, $w(x)$, in the $z$-direction, as a function of the $x$-direction along the axes of rolls and the strip. Since not all rolls are coincident vertically along the $y$-axis, the horizontal displacement is non-zero. Figures 19c and 19e illustrate the contact force distribution at the interface between the strip and the work roll, and between the other various rolls. Like that observed for the 4-High mill, in the absence of any strip profile control devices, the contact force between the strip and the work roll increases in the vicinity of the strip edges, leading to the “natural” strip crown. An interesting and useful characteristic of the 20-High mill is its ability to laterally transfer much of the vertical roll bite load. This is evidenced when comparing the general magnitude contact force between the second intermediate driver roll (DRVR) and backing bearings A and B, respectively (BRG A, BRG B). Table 6 summarizes the results for the 20-High mill simulation. The C25 strip crown is 0.0605 mm, since the upper half thickness is 0.0302 mm greater at the strip center than at the C25 edge locations. This crown corresponds to 6.675% of the exit thickness at the strip center.

Table 6 – Results summary for application of new model to 20-High mill

<table>
<thead>
<tr>
<th>New Model Results Summary</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strip center thickness, $h$ (mm)</td>
<td>0.9063</td>
</tr>
<tr>
<td>Strip C25 thickness, $h_{25}$ (mm)</td>
<td>0.8458</td>
</tr>
<tr>
<td>Strip crown, C25 (mm)</td>
<td>0.0605</td>
</tr>
<tr>
<td>Strip crown, C25 (%)</td>
<td>6.675</td>
</tr>
<tr>
<td>Total force, F (MN)</td>
<td>2.292</td>
</tr>
</tbody>
</table>
Figure 19 – 252 element model results for 20-high Sendzimir rolling mill

a) vertical displacement distribution, \( v(x) \)
b) horizontal displacement distribution, \( w(x) \)
c) contact force distribution
d) thickness relative to edge for upper half of strip
Figure 19d illustrates the cross sectional thickness of the upper half of the strip relative to that of the strip edge. The increased rigidity of the 20-High mill, in comparison to the 4-High mill, causes it to “flatten” the natural strip profile over a majority of the strip width, but significant edge-drop is still present. The tendency to create such a large edge-drop leads most users of 20-High mills to decrease the diameters of the first intermediate rolls near their ends. Shifting of these tapered first intermediate rolls provides control of the force distribution near the strip edges and hence increases control over the magnitude of edge-drop. The new method is fully extendable to include the effects of shifting tapered rolls, in addition to the effects of roll bending mechanisms on 20-High mills.

Figure 20 illustrates the rapid convergence of the center and edge displacements of the strip in a 20-High mill with respect to the number of Timoshenko beam elements.

![Figure 20](image_url)

**Figure 20 – Mesh convergence study with presented model for 20-high rolling mill**
3.12 Dynamic Deflection Model

A major advantage of the proposed method for modeling rolling mill deflection is the ability to predict the dynamic response and associated vibration characteristics of rolling mills. By constructing a global mass matrix, [M], in addition to the linearized global stiffness matrix, [K], the standard eigenvalue problem can be solved to obtain the natural frequencies and mode shapes of vibration. Natural frequencies are important in mill design to avoid excessive vibration, such as mill “chattering,” and to prevent structural failure. Because of linearization, the natural mode shapes of vibration at a given static loading condition can be obtained by superposition of the no-load mode shapes with the statically determined displacements. Using widely known methods, the response to harmonic loading, response history, and spectral response of the rolling mill structure can be readily obtained using the proposed global stiffness matrix, [K], the global mass matrix, [M], and for some cases, a user-defined global damping matrix, [C] [44].
4. VALIDATION OF THE STATIC MODEL USING COMMERCIAL FINITE ELEMENT ANALYSIS

4.1 Validation for a 4-High Rolling Mill

To evaluate the ability of the new model to accurately predict the deflection behavior of the 4-High mill in Section 3.11, a comparison of the vertical displacement results was made using the commercial Finite Element Analysis (FEA) package ABAQUS version 6.6-1. Due to the computational expense of contact-type structural analyses in conventional FEA, all planes of symmetry for the rolling mill were exploited, leading to the 1/8th model of the 4-High mill shown in Figure 21. Over 64,000 three-dimensional tetrahedral elements were generated as a result of the extreme mesh

Figure 21 – 1/8th symmetric ABAQUS FEA model of 4-High mill (64,054 3D tetrahedral elements)
refinement assigned automatically by ABAQUS at the contact interfaces between the rolls and strip.

Rather than performing elastic-plastic FEA, in order to obtain a direct comparison and validate the new model, it was decided to assign elastic parameters to the strip elements in the ABAQUS FEA model such that they represent a one-dimensional linear elastic foundation. This was accomplished by assigning specific values to the Poisson ratio, \( \nu \), and the Young’s (elastic) modulus, \( E \) of the strip. If a constant foundation modulus, \( k(x) = \beta \), is assumed and one notes that this modulus is equivalent to the spring constant per unit strip width \( w \), the following expression for an area modulus \( \beta_A \), can be written where \( \beta_A = \beta/b \), and \( A = bw \) is the contact area between the strip and the work roll.

\[
\beta_A = \frac{F/\Delta y}{A} \tag{21}
\]

In Equation 21, \( F \) is the total load applied to the strip, \( \Delta y \) is the foundation displacement (strip thickness reduction), and \( A \) is the foundation area. Hooke’s law for the y-direction strain, which corresponds to the strip thickness reduction, is:

\[
\varepsilon_y = \frac{1}{E} \left[ \sigma_y - \nu (\sigma_x + \sigma_z) \right] \tag{22}
\]

In Equation 22, \( \varepsilon_y \) is the true strain, \( E \) is the elastic modulus, \( \nu \) is the Poisson ratio, and \( \sigma_x, \sigma_y, \) and \( \sigma_z \) are the average orthogonal stress components. Next, the Poisson ratio is
assigned to zero in order to achieve one-dimensional behavior of the strip foundation, and
the true strain, $\varepsilon_y$, is written in terms of the ratio of thickness reduction, which is simply
the engineering strain. Equation 22 can therefore be written as:

$$\ln(1 + \Delta y / H) = \frac{\sigma_y}{E}$$  \hspace{1cm} (23)

In Equation 23, $H$ is the initial strip thickness. Since average stress $\sigma_y$ is equal to $F/A$, the total force $F$ from Equation 21 can be substituted into Equation 23 and rearranged to obtain an expression for the elastic modulus, $E$, in terms of the specified area foundation modulus, $\beta_A$:

$$E = \frac{\beta_A \Delta y}{\ln(1 + \Delta y / H)}$$  \hspace{1cm} (24)

The engineering strain may be used directly to obtain Equation 25 if the strip thickness reduction is less than about ten percent:

$$E = \beta_A \, H$$  \hspace{1cm} (25)

To validate the new model with the results of the ABAQUS FEA model, the three-dimensional tetrahedral strip elements of the FEA model are assigned a zero Poisson ratio and an equivalent elastic modulus, $E$, using Equations 24 or 25. To determine the equivalent elastic modulus for the strip upper half section, half the initial strip thickness and half the thickness reduction, but twice the foundation modulus are
used in Equations 24 and 25. Substituting the data for the 4-High mill of Section 3.11 into Equation 24, and estimating contact dimension $b$ using Equation 26 for rigid rolls [1], an equivalent approximate strip elastic modulus, $E = 15300 \text{ N/mm}^2$, is obtained for the tetrahedral elements of the strip upper half section. Note that because of the strip crown phenomenon, the strip thickness reduction, $\Delta y$, in Equation 26 represents a nominal reduction over the width of the strip in order to obtain a nominal contact dimension $b$:

$$b = \sqrt{\frac{D_w \Delta y}{2}} \quad (26)$$

An amplified view of the results of the vertical displacement field for the ABAQUS FEA model is shown in Figure 22. The same boundary conditions and material properties that were used for the new model of 4-High mill in Section 3.11 were applied here. The typical displacement pattern at the interface between the strip and the work roll, leading to the strip crown phenomenon, is readily observable in Figure 22.

Plots of the displacement of the axes of the work roll and backup roll in addition to plots of displacement at the contact interfaces between the rolls and the strip are provided in Figure 23. Also shown in Figure 23 is a plot of the predicted displacement of the interface between the work roll and the strip based on the new model.
Figure 22 – Vertical displacement of 4-High mill using ABAQUS FEA

Figure 23 – Vertical displacement of roll axes and strip upper surface in 4-High rolling mill
Since the strip profile and corresponding crown are obtained from the displacement field, a direct evaluation of the performance of the new model can be made. Table 7 provides a numerical comparison between the displacements predicted using the new model and those obtained for three iterations of automatic mesh assignment using the ABAQUS FEA model. Note that both the displacement field contour plot of Figure 22 and the FEA displacement curves of Figure 23 are based on the third iteration.

Table 8 indicates the error in the displacement predicted by the new model relative to the ABAQUS results. It is evident that, while the new model comprises only 48 Timoshenko beam elements and associated elastic foundations, it is able to predict very accurate displacements relative to a conventional FEA elastic contact analysis using 64,054 elements. Displacements at the strip center, C25 location, and strip edge are predicted to within 1.35%, 1.20%, and 2.73%, respectively, of the values computed for the third iteration of the ABAQUS model. Furthermore, Table 8 indicates an overall convergence trend of the FEA results toward those of the new model, providing additional validation.

Table 7 – Vertical displacement comparison between ABAQUS FEA and new model

<table>
<thead>
<tr>
<th>Model</th>
<th>No. Elements</th>
<th>Strip Center Disp. (mm)</th>
<th>Strip C25 Disp. (mm)</th>
<th>Strip Edge Disp. (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>FEA iter 1</td>
<td>44716</td>
<td>4.3896</td>
<td>3.7315</td>
<td>3.3650</td>
</tr>
<tr>
<td>FEA iter 2</td>
<td>42672</td>
<td>4.2718</td>
<td>3.6640</td>
<td>3.3147</td>
</tr>
<tr>
<td>FEA iter 3</td>
<td>64054</td>
<td>4.2459</td>
<td>3.5884</td>
<td>3.2408</td>
</tr>
<tr>
<td>New Model</td>
<td>48</td>
<td>4.1891</td>
<td>3.6316</td>
<td>3.1521</td>
</tr>
</tbody>
</table>
Table 8 – Vertical displacement error of new model relative to ABAQUS FEA

<table>
<thead>
<tr>
<th>Model</th>
<th>No. Elements</th>
<th>Center Disp. Error (%)</th>
<th>C25 Disp. Error (%)</th>
<th>Edge Disp. Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>FEA iter 1</td>
<td>44716</td>
<td>-4.59</td>
<td>-2.68</td>
<td>-6.33</td>
</tr>
<tr>
<td>FEA iter 2</td>
<td>42672</td>
<td>-1.95</td>
<td>-0.88</td>
<td>-4.90</td>
</tr>
<tr>
<td>FEA iter 3</td>
<td>64054</td>
<td>-1.35</td>
<td>1.20</td>
<td>-2.73</td>
</tr>
</tbody>
</table>

Although it was intended to simulate the deflection behavior of the 20-high Sendzimir rolling mill using ABAQUS, convergence difficulties in the multi-contact problem precluded any solution with acceptable convergence. This was the case even for solid backing rolls (Figure 24) rather than segmented bearing-shaft rolls as in the actual mill. While this circumstance lends some validity to any other model that is capable of efficiently realizing a solution, no comparison for 20-High mills was made with respect to the results of the new model presented in this work.

![Figure 24 – Preliminary ABAQUS FEA validation model for 20-high rolling mill](image)
4.2 Evaluation of Elastic Foundations using FEA and Design of Experiments (DOE)

To more accurately predict the roll elastic foundation moduli a study of the three-dimensional deflection behavior of two rolls in lengthwise contact using FEA and Design of Experiments (DOE) was examined. The study investigated the influence of roll diameters, roll length, unit contact force, and friction coefficient on the center-to-center and center-to-surface displacements of two cylindrical rolls. Insight into how the elastic foundation moduli, \( k(x) \), \( k_1(x) \), and \( k_2(x) \), depicted in Figure 25, vary as a function of axial position \( x \) along the roll axes is provided.

![Figure 25 – 1/8 symmetric model to study the effects of geometry, friction, and loading on the elastic foundation moduli between rolls](image-url)
Since many of the classical analytical solutions for the deflection-load relationship between rolls (discussed earlier in Section 3.4) assume either infinite length rolls (plane-strain solution) or both infinite length and infinite diameter rolls (half-space solution) under frictionless conditions, it was decided to investigate the effects of finite roll geometry, friction, and loading magnitude on the elastic foundation moduli using DOE and FEA. The study involved a series of experiments involving the two rolls, “Roll 1” and “Roll 2,” in which the five parameters examined included the axial position $x$, the radius of Roll 1, $R_1$, the length of both rolls, $L$, the force per unit length between the rolls, $F$, and the interfacial friction coefficient, $\mu$.

Figure 26 – 1/8th symmetric model showing roll radii and length dimensions
The levels of the five parameters, denoted $X_i$ for $i = 1$ to 5, were modified according to a $5^{th}$ order orthogonal Central Composite Design (CCD) [49]. Orthogonality avoids confounding of the individual parameter effects and assures that their levels of significance are independently identified. The $5^{th}$ order CCD requires a total of 43 design points in order to fit $2^{nd}$ order response surface polynomials, for the dependent variables $k(x)/k_0$, $k_1(x)/k_{10}$, $k_2(x)/k_{20}$, in Equation 27a-c.

$$
\frac{k(x)}{k_0} = \beta_0 + \sum_{i=1}^{k} \beta_i X_i + \sum_{i=1}^{k} \beta_{ii} X_i^2 + \sum_{i=1}^{k} \sum_{j=i+1}^{k} \beta_{ij} X_i X_j
$$

(27a)

$$
\frac{k_1(x)}{k_{10}} = \beta_0 + \sum_{i=1}^{k} \beta_i X_i + \sum_{i=1}^{k} \beta_{ii} X_i^2 + \sum_{i=1}^{k} \sum_{j=i+1}^{k} \beta_{ij} X_i X_j
$$

(27b)

$$
\frac{k_2(x)}{k_{20}} = \beta_0 + \sum_{i=1}^{k} \beta_i X_i + \sum_{i=1}^{k} \beta_{ii} X_i^2 + \sum_{i=1}^{k} \sum_{j=i+1}^{k} \beta_{ij} X_i X_j
$$

(27c)

The dependent variables $k(x)/k_0$, $k_1(x)/k_{10}$, $k_2(x)/k_{20}$ represent the ratio of the foundation moduli at axial position $x$ to the respective foundation moduli at the midpoint of the rolls ($x = 0$), which would most closely represent plane-strain or half-space assumptions. The definitions for the original and non-dimensional response surface variables $X_i$ are also shown in Table 9. The domain of these variables is designed to accommodate the envelope of possible rolling conditions for various types of mills.
Table 9 – Definition of variables for Central Composite DOE

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Original Variables</th>
<th>Variable</th>
<th>Min.</th>
<th>Max.</th>
</tr>
</thead>
<tbody>
<tr>
<td>X1</td>
<td>Radius of Roll 1 (in.):</td>
<td>R1</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>X2</td>
<td>Radius of Roll 2 (in.):</td>
<td>R2</td>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>X3</td>
<td>Half Length (in.):</td>
<td>L</td>
<td>5</td>
<td>75</td>
</tr>
<tr>
<td>X4</td>
<td>Contact Force Per Length (lb/in):</td>
<td>F</td>
<td>200</td>
<td>20000</td>
</tr>
<tr>
<td>X5</td>
<td>Friction Coef.</td>
<td>u</td>
<td>0</td>
<td>0.4</td>
</tr>
<tr>
<td></td>
<td>Axial Position along Roll (in):</td>
<td>x</td>
<td>0</td>
<td>L</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Non-Dimensional Variables</th>
<th>Variable</th>
<th>Min.</th>
<th>Max.</th>
</tr>
</thead>
<tbody>
<tr>
<td>X1</td>
<td>Ratio of R2 to R1:</td>
<td>R2 / R1</td>
<td>0.2</td>
<td>1</td>
</tr>
<tr>
<td>X2</td>
<td>Ratio of Half Length to R1:</td>
<td>L / R1</td>
<td>1</td>
<td>15</td>
</tr>
<tr>
<td>X3</td>
<td>Ratio of Force Per Length to (R1 E):</td>
<td>F / (R1 E)</td>
<td>1.33E-06</td>
<td>0.000133</td>
</tr>
<tr>
<td>X4</td>
<td>Friction Coef.</td>
<td>u</td>
<td>0</td>
<td>0.4</td>
</tr>
<tr>
<td>X5</td>
<td>Ratio of Axial Pos. to Half Length</td>
<td>x / L</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 10 illustrates the coded, non-dimensional, and original variable values for each of the 25 runs of the orthogonal Central Composite Design. Linear regression was executed using the coded variables to generate the response surfaces in Equations 27a-c.

Table 10 – Values for coded, non-dimensional, and original CCD variables

**CENTRAL COMPOSITE EXPERIMENTAL DESIGN**

<table>
<thead>
<tr>
<th>CODED VARIABLES</th>
<th>NON-DIMENSIONAL VARIABLES</th>
<th>ORIGINAL VARIABLES</th>
</tr>
</thead>
<tbody>
<tr>
<td>Run No. X1 X2 X3 X4</td>
<td>Run No. X1 X2 X3 X4</td>
<td>Run No. X1 X2 X3 X4</td>
</tr>
<tr>
<td>1 -1 -1 -1</td>
<td>1 0.349074 3.614053 2.6E-05 0.074687</td>
<td>1 1.746872 18.07027 3897.018 0.074687</td>
</tr>
<tr>
<td>2 1 -1 -1</td>
<td>2 0.850626 3.614053 2.6E-05 0.074687</td>
<td>2 4.253128 18.07027 3897.018 0.074687</td>
</tr>
<tr>
<td>3 -1 -1 -1</td>
<td>3 0.850626 3.614053 2.6E-05 0.074687</td>
<td>3 1.746872 18.07027 3897.018 0.074687</td>
</tr>
<tr>
<td>4 1 1 -1</td>
<td>4 0.850626 3.614053 2.6E-05 0.074687</td>
<td>4 4.253128 18.07027 3897.018 0.074687</td>
</tr>
<tr>
<td>5 -1 1 -1</td>
<td>5 0.850626 3.614053 0.000109 0.074687</td>
<td>5 1.746872 18.07027 16302.98 0.074687</td>
</tr>
<tr>
<td>6 1 1 1</td>
<td>6 0.850626 3.614053 0.000109 0.074687</td>
<td>6 4.253128 18.07027 16302.98 0.074687</td>
</tr>
<tr>
<td>7 -1 1 1</td>
<td>7 0.850626 3.614053 0.000109 0.074687</td>
<td>7 1.746872 18.07027 16302.98 0.074687</td>
</tr>
<tr>
<td>8 1 1 -1</td>
<td>8 0.850626 3.614053 0.000109 0.074687</td>
<td>8 4.253128 18.07027 16302.98 0.074687</td>
</tr>
<tr>
<td>9 -1 -1 -1</td>
<td>9 0.850626 3.614053 0.000109 0.074687</td>
<td>9 1.746872 18.07027 3897.018 0.074687</td>
</tr>
<tr>
<td>10 1 -1 -1</td>
<td>10 0.850626 3.614053 0.000109 0.074687</td>
<td>10 4.253128 18.07027 3897.018 0.074687</td>
</tr>
<tr>
<td>11 -1 1 -1</td>
<td>11 0.850626 3.614053 0.000109 0.074687</td>
<td>11 1.746872 18.07027 3897.018 0.074687</td>
</tr>
<tr>
<td>12 1 1 1</td>
<td>12 0.850626 3.614053 0.000109 0.074687</td>
<td>12 4.253128 18.07027 3897.018 0.074687</td>
</tr>
<tr>
<td>13 -1 1 1</td>
<td>13 0.850626 3.614053 0.000109 0.074687</td>
<td>13 1.746872 18.07027 16302.98 0.074687</td>
</tr>
<tr>
<td>14 1 1 1</td>
<td>14 0.850626 3.614053 0.000109 0.074687</td>
<td>14 4.253128 18.07027 16302.98 0.074687</td>
</tr>
<tr>
<td>15 -1 1 1</td>
<td>15 0.850626 3.614053 0.000109 0.074687</td>
<td>15 1.746872 18.07027 16302.98 0.074687</td>
</tr>
<tr>
<td>16 1 1 1</td>
<td>16 0.850626 3.614053 0.000109 0.074687</td>
<td>16 4.253128 18.07027 16302.98 0.074687</td>
</tr>
<tr>
<td>17 0 0 0 0</td>
<td>17 0.6 8 6.7E-05 0.2</td>
<td>17 3 40 10100 0.2</td>
</tr>
<tr>
<td>18 0 1.596007 0 0</td>
<td>18 0.325313 0 0</td>
<td>18 1 40 10100 0.2</td>
</tr>
<tr>
<td>19 0 1.596007 0 0</td>
<td>19 0 0 0 0.074687</td>
<td>19 5 40 10100 0.2</td>
</tr>
<tr>
<td>20 0 1.596007 0 0</td>
<td>20 0.6 1 6.7E-05 0.2</td>
<td>20 3 5 10100 0.2</td>
</tr>
<tr>
<td>21 0 1.596007 0 0</td>
<td>21 0.6 15 6.7E-05 0.2</td>
<td>21 3 75 10100 0.2</td>
</tr>
<tr>
<td>22 0 1.596007 0 0</td>
<td>22 0.6 8 6.7E-05 0.2</td>
<td>22 3 40 2000 0.2</td>
</tr>
<tr>
<td>23 0 1.596007 0 0</td>
<td>23 0.6 8 0.000133 0.2</td>
<td>23 3 40 20000 0.2</td>
</tr>
<tr>
<td>24 0 1.596007 0 0</td>
<td>24 0.6 8 6.7E-05 0 0</td>
<td>24 3 40 10100 0</td>
</tr>
<tr>
<td>25 0 1.596007 0 0</td>
<td>25 0.6 8 6.7E-05 0.4</td>
<td>25 3 40 10100 0.4</td>
</tr>
</tbody>
</table>

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To date, no other rolling studies have been published for an investigation of this type that includes two three-dimensional rolls in contact with a frictional interaction between them.

Although the 5th order Central Composite Design requires 43 orthogonal design points to fit the response surface in Equations 27a-c, as shown in Table 10 only 25 unique ABAQUS FEA runs were required because more than one design point for the axial position variable, $X_5$, was obtained from the same FEA model. A typical result illustrating the vertical deflection of the rolls used to determine the foundation moduli is shown in Figure 27a and in an expanded view in Figure 27b.
The contour plots of Figures 27a and 27b suggest that a change in the vertical displacement field develops as a function of the axial position along the rolls even though the same loading conditions exist throughout. Similar plots for other runs of the Central Composite Design indicate that this phenomenon is more pronounced as the length-to-diameter ratio increases and implies that the endpoint of the rolls (at $x = L/2$) exhibits manifest behavior away from that of a plane-strain condition and toward that of a plane-stress condition. Since the elastic foundation moduli $k(x)$, $k_1(x)$, and $k_2(x)$ depend directly on the relationship between the load and displacement between the roll axes, Equations 27a-c are fitted using the corresponding FEA displacement data.
An Analysis of Variance (ANOVA) to determine the statistical significance of each of the individual polynomial terms in the response surfaces of Equations 27a-c is provided in Tables 11a-c [49]. Each table indicates the relevance of the various factors on the elastic foundation moduli \( k(x) \), \( k_1(x) \), and \( k_2(x) \) at 10%, 5%, and 1% significance levels, corresponding to 90%, 95%, and 99% confidence levels respectively. The response surface polynomial terms include the linear, quadratic, and mixed quadratic (interaction) effects of the general parameters (axial position, roll radius, roll length, contact force per unit length, and friction coefficient) on the elastic foundation moduli.

In each of Tables 11a, 11b, and 11c, it is evident that the foundation moduli \( k(x) \), \( k_1(x) \), and \( k_2(x) \) at an arbitrary position \( x \) along the roll are closely related to the corresponding moduli at the midpoints of the rolls \( k_0, k_{10}, \) and \( k_{20} \). This is evidenced by the quantity of the regression sum of squares for the constant terms \( \beta_0X_0 \) in Tables 11a-c, which are several orders of magnitude greater than the regression sum of squares for the other terms of each response surface. Nevertheless, several other terms are still statistically significant. Table 11a indicates that at the 10% significance (90% confidence) level, the significant terms for \( k(x) \) include the linear effects of roll length, friction coefficient, and axial position. In addition, second-order interaction effects exist, involving the radius of Roll 2 and the roll lengths, the contact force and the roll lengths, and the axial position and the roll lengths. At the 5% significance (95% confidence) level, the linear effect of friction coefficient, and the interaction effects between the radius of Roll 2 and the roll lengths, and between the contact force and the roll lengths
are removed. The factors at the 1% significance (99% confidence) level are identical to factors at the 5% significance (95% confidence) level.

Table 11b indicates that at the 10% significance (90% confidence) level, the significant terms for \( k_1(x) \) include first-order effects of only the roll length. Second-order interactions exist between the contact force and the roll lengths, and between the axial position and the roll lengths. At the 5% significance (95% confidence) level, the interaction effect between the contact force and the roll lengths is removed. At the 1% significance (99% confidence) level, the interaction effect between the axial position and the roll lengths is not significant.

Table 11c shows that at both the 10% significance (90% confidence) level and the 5% significance (95% confidence) level, the significant terms for \( k_2(x) \) include the linear effects of Roll 2 radius, roll lengths, and axial position. Second-order effects of Roll 2 radius and roll lengths are also present. Interaction effects occur between the radius of Roll 2 and the roll lengths, and between the axial position and the roll lengths. At the 1% significance (99% confidence) level, the remaining significant factors include only the first-order effect of the roll lengths and the second-order interaction effect between the roll lengths and the axial position.

The difference in the significance of terms for \( k_1(x) \) and \( k_2(x) \) is due simply to the fact that the radius of Roll 1 was fixed, and therefore no terms involving the radius could appear. It was thus decided to employ the response surface for \( k_2(x) \) to modify the
nominal elastic foundation moduli between the axis and the surface of a given roll in addition to the response surface for $k(x)$ to modify the foundation moduli between the axes of two rolls. It may have been better to include changes to both radii in the Design of Experiments, but this would have required far greater number of analyses and extensive computation time.

Table 11a – Analysis of Variance Table for $k(x)$

<table>
<thead>
<tr>
<th>Source (Eqn. Term)</th>
<th>Sum of Squares</th>
<th>Degrees of Freedom</th>
<th>Mean Square</th>
<th>F-Test Value</th>
<th>10% Sig. Value</th>
<th>5% Sig. Value</th>
<th>1% Sig. Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>B0 X0</td>
<td>4.279E+01</td>
<td>1</td>
<td>4.279E+01</td>
<td>3.485E+06</td>
<td>SIG</td>
<td>SIG</td>
<td>SIG</td>
</tr>
<tr>
<td>B1 x1</td>
<td>7.144E+06</td>
<td>1</td>
<td>7.144E+06</td>
<td>5.818E+01</td>
<td>SIG</td>
<td>SIG</td>
<td>SIG</td>
</tr>
<tr>
<td>B2 x2</td>
<td>7.889E-04</td>
<td>1</td>
<td>7.889E-04</td>
<td>6.425E+01</td>
<td>SIG</td>
<td>SIG</td>
<td>SIG</td>
</tr>
<tr>
<td>B3 x3</td>
<td>1.385E-05</td>
<td>1</td>
<td>1.385E-05</td>
<td>1.128E+00</td>
<td>SIG</td>
<td>SIG</td>
<td>SIG</td>
</tr>
<tr>
<td>B4 x4</td>
<td>4.010E-05</td>
<td>1</td>
<td>4.010E-05</td>
<td>3.266E+00</td>
<td>SIG</td>
<td>SIG</td>
<td>SIG</td>
</tr>
<tr>
<td>B5 x5</td>
<td>2.203E-04</td>
<td>1</td>
<td>2.203E-04</td>
<td>1.794E+01</td>
<td>SIG</td>
<td>SIG</td>
<td>SIG</td>
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<tr>
<td>B11 x11</td>
<td>1.760E-05</td>
<td>1</td>
<td>1.760E-05</td>
<td>1.433E+00</td>
<td>SIG</td>
<td>SIG</td>
<td>SIG</td>
</tr>
<tr>
<td>B22 x22</td>
<td>1.478E-05</td>
<td>1</td>
<td>1.478E-05</td>
<td>1.204E+00</td>
<td>SIG</td>
<td>SIG</td>
<td>SIG</td>
</tr>
<tr>
<td>B33 x33</td>
<td>7.566E-06</td>
<td>1</td>
<td>7.566E-06</td>
<td>6.162E-01</td>
<td>SIG</td>
<td>SIG</td>
<td>SIG</td>
</tr>
<tr>
<td>B44 x44</td>
<td>4.546E-06</td>
<td>1</td>
<td>4.546E-06</td>
<td>3.702E-01</td>
<td>SIG</td>
<td>SIG</td>
<td>SIG</td>
</tr>
<tr>
<td>B55 x55</td>
<td>2.140E-05</td>
<td>1</td>
<td>2.140E-05</td>
<td>1.743E+00</td>
<td>SIG</td>
<td>SIG</td>
<td>SIG</td>
</tr>
<tr>
<td>B12 x12</td>
<td>4.827E-05</td>
<td>1</td>
<td>4.827E-05</td>
<td>3.931E+00</td>
<td>SIG</td>
<td>SIG</td>
<td>SIG</td>
</tr>
<tr>
<td>B13 x13</td>
<td>4.511E-09</td>
<td>1</td>
<td>4.511E-09</td>
<td>3.674E-04</td>
<td>SIG</td>
<td>SIG</td>
<td>SIG</td>
</tr>
<tr>
<td>B14 x14</td>
<td>1.104E-05</td>
<td>1</td>
<td>1.104E-05</td>
<td>8.992E-01</td>
<td>SIG</td>
<td>SIG</td>
<td>SIG</td>
</tr>
<tr>
<td>B15 x15</td>
<td>1.203E-05</td>
<td>1</td>
<td>1.203E-05</td>
<td>9.799E-01</td>
<td>SIG</td>
<td>SIG</td>
<td>SIG</td>
</tr>
<tr>
<td>B23 x23</td>
<td>4.506E-05</td>
<td>1</td>
<td>4.506E-05</td>
<td>3.670E+00</td>
<td>SIG</td>
<td>SIG</td>
<td>SIG</td>
</tr>
<tr>
<td>B24 x24</td>
<td>6.504E-06</td>
<td>1</td>
<td>6.504E-06</td>
<td>5.297E-01</td>
<td>SIG</td>
<td>SIG</td>
<td>SIG</td>
</tr>
<tr>
<td>B25 x25</td>
<td>7.407E-04</td>
<td>1</td>
<td>7.407E-04</td>
<td>6.032E+01</td>
<td>SIG</td>
<td>SIG</td>
<td>SIG</td>
</tr>
<tr>
<td>B34 x34</td>
<td>1.083E-06</td>
<td>1</td>
<td>1.083E-06</td>
<td>8.827E-02</td>
<td>SIG</td>
<td>SIG</td>
<td>SIG</td>
</tr>
<tr>
<td>B35 x35</td>
<td>1.018E-05</td>
<td>1</td>
<td>1.018E-05</td>
<td>8.296E-01</td>
<td>SIG</td>
<td>SIG</td>
<td>SIG</td>
</tr>
<tr>
<td>B45 x45</td>
<td>1.070E-05</td>
<td>1</td>
<td>1.070E-05</td>
<td>6.714E-01</td>
<td>SIG</td>
<td>SIG</td>
<td>SIG</td>
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<tr>
<td>Error</td>
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<td>1.227E-05</td>
<td>1.019E+00</td>
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<tr>
<td>Total</td>
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<td>1.019E+00</td>
<td>1.019E+00</td>
<td>SIG</td>
<td>SIG</td>
<td>SIG</td>
</tr>
</tbody>
</table>
### Table 11b – Analysis of Variance Table for $k_1(x)$

<table>
<thead>
<tr>
<th>Source (Eqn. Term)</th>
<th>Sum of Squares</th>
<th>Degrees of Freedom</th>
<th>Mean Square</th>
<th>F-Test Value</th>
<th>10% Sig. Level</th>
<th>5% Sig. Level</th>
<th>1% Sig. Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>B0 x0</td>
<td>4.3227E+01</td>
<td>1</td>
<td>4.3227E+01</td>
<td>4.4631E+05</td>
<td>SIG</td>
<td>SIG</td>
<td>SIG</td>
</tr>
<tr>
<td>B1 x1</td>
<td>1.1524E-04</td>
<td>1</td>
<td>1.1524E-04</td>
<td>1.1951E+00</td>
<td>SIG</td>
<td>SIG</td>
<td>SIG</td>
</tr>
<tr>
<td>B2 x2</td>
<td>7.8942E-04</td>
<td>1</td>
<td>7.8942E-04</td>
<td>8.1871E+00</td>
<td>SIG</td>
<td>SIG</td>
<td>SIG</td>
</tr>
<tr>
<td>B3 x3</td>
<td>2.3263E-04</td>
<td>1</td>
<td>2.3263E-04</td>
<td>2.4126E+00</td>
<td>SIG</td>
<td>SIG</td>
<td>SIG</td>
</tr>
<tr>
<td>B4 x4</td>
<td>4.7172E-11</td>
<td>1</td>
<td>4.7172E-11</td>
<td>4.8922E-07</td>
<td>SIG</td>
<td>SIG</td>
<td>SIG</td>
</tr>
<tr>
<td>B5 x5</td>
<td>7.4391E-05</td>
<td>1</td>
<td>7.4391E-05</td>
<td>7.7150E-01</td>
<td>SIG</td>
<td>SIG</td>
<td>SIG</td>
</tr>
<tr>
<td>B11 x11</td>
<td>5.2735E-05</td>
<td>1</td>
<td>5.2735E-05</td>
<td>5.4691E-01</td>
<td>SIG</td>
<td>SIG</td>
<td>SIG</td>
</tr>
<tr>
<td>Error</td>
<td>2.0249E-03</td>
<td>21</td>
<td>9.6423E-05</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>4.3232E+01</td>
<td>42</td>
<td>1.0293E+00</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Table 11c – Analysis of Variance Table for $k_2(x)$

<table>
<thead>
<tr>
<th>Source (Eqn. Term)</th>
<th>Sum of Squares</th>
<th>Degrees of Freedom</th>
<th>Mean Square</th>
<th>F-Test Value</th>
<th>10% Sig. Level</th>
<th>5% Sig. Level</th>
<th>1% Sig. Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>B0 x0</td>
<td>4.2332E-01</td>
<td>1</td>
<td>4.2332E-01</td>
<td>5.0445E+05</td>
<td>SIG</td>
<td>SIG</td>
<td>SIG</td>
</tr>
<tr>
<td>B1 x1</td>
<td>1.1524E-04</td>
<td>1</td>
<td>1.1524E-04</td>
<td>1.1951E+00</td>
<td>SIG</td>
<td>SIG</td>
<td>SIG</td>
</tr>
<tr>
<td>B2 x2</td>
<td>7.8942E-04</td>
<td>1</td>
<td>7.8942E-04</td>
<td>8.1871E+00</td>
<td>SIG</td>
<td>SIG</td>
<td>SIG</td>
</tr>
<tr>
<td>B3 x3</td>
<td>2.3263E-04</td>
<td>1</td>
<td>2.3263E-04</td>
<td>2.4126E+00</td>
<td>SIG</td>
<td>SIG</td>
<td>SIG</td>
</tr>
<tr>
<td>B4 x4</td>
<td>4.7172E-11</td>
<td>1</td>
<td>4.7172E-11</td>
<td>4.8922E-07</td>
<td>SIG</td>
<td>SIG</td>
<td>SIG</td>
</tr>
<tr>
<td>B5 x5</td>
<td>7.4391E-05</td>
<td>1</td>
<td>7.4391E-05</td>
<td>7.7150E-01</td>
<td>SIG</td>
<td>SIG</td>
<td>SIG</td>
</tr>
<tr>
<td>B11 x11</td>
<td>5.2735E-05</td>
<td>1</td>
<td>5.2735E-05</td>
<td>5.4691E-01</td>
<td>SIG</td>
<td>SIG</td>
<td>SIG</td>
</tr>
<tr>
<td>Error</td>
<td>2.0249E-03</td>
<td>21</td>
<td>9.6423E-05</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>4.2340E+01</td>
<td>42</td>
<td>1.0081E+00</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

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5. OPTIMIZATION AND RELIABILITY ANALYSIS EXAMPLES

The presented method to predict the strip thickness profile that evolves as a result of the deflection of the various components within a rolling mill is useful in optimizing many tasks in the rolling process. Three important operational tasks that facilitate desired strip thickness profile and corresponding flatness include: i) pass-schedule optimization to assign the most suitable gauge reduction schedule, ii) optimization of the diameter-profiles ground onto the rolls, and iii) optimization of flatness control actuators. In sections 5.1 and 5.2, examples of pass-schedule optimization and roll diameter profile optimization are given.

Although no example of flatness actuator optimization is provided in this work, the presented method lends itself well to this type of on-line control system since it involves repeated calculation of the strip profile (and corresponding flatness) based on perturbations of the flatness actuator displacements at a nominal mill loading condition. The matrix inversion method of solution for the presented method (described in Section 3.6) provides a means to rapidly obtain perturbed load vector results, allowing direct application of the introduced method to accommodate flatness actuator optimization. Numerous examples of strip flatness control algorithms for rolling mills, including the more complex cluster-type mills, are available in the literature [e.g. 54]. But since accurate and rapid physics-based models of the rolling mills are not abundant,
particularly for cluster mills, many of the control-system methods employ empirically-determined gain matrices to relate the strip profile actuators and sensors. Other control-system methods that do employ physics-based static models frequently use gain matrices based on only a few mill operating conditions. Experienced mill operating personnel, however, are quick to note that the effect of profile and flatness control actuators varies with the gauge and width of the strip, its mechanical properties, and the configuration of the rolls. Hence, value exists in incorporating the strip profile model developed here to design more effective on-line flatness control systems.

**Reliability Analysis for Strip Profile and Flatness**

An area of study that is beneficial to manufacturing, and which may be incorporated into both on-line and off-line optimization routines when combined with sufficient industry data, is that of Reliability Analysis. The field of Reliability Analysis is concerned with uncertainties or random input parameters in a system, and the resulting uncertainty of some dependent property or parameter that, upon reaching certain threshold, may lead to failure in the system. Reliability studies have been reinvigorated in recent years due to the availability of automated sources of detailed manufacturing data and the introduction of new methods to calculate the probability of failure or success for complex problems. New techniques to execute reliability studies have been developed in recent years, as outlined by Halder and Mahadevan [50]. Despite this, little work has been published on the use of reliability-based methods to optimize the rolling process and improve the overall dimensional quality of the rolled strip. The probability of achieving a strip profile and corresponding flatness within specified limits (to prevent,
for example, the occurrence of edge-waves and center-buckles in rolled strip) is an area of research that has important manufacturing implications. The rejection and scrapping of rolled material is a common occurrence, particularly during the start-up of new rolling mills when activities involving trial and error can be significant. As a result, in Sections 5.3 an example of predicting the reliability of achieving the desired strip crown and flatness is given using estimated random data distributions for some factors affecting the profile and flatness. The resultant reliability calculation may easily be incorporated into mathematical optimization routines as either a constraint or an objective function.

5.1 Rolling Pass Schedule Optimization for a 20-High Mill

A rolling pass-schedule consists of assigning a series of thickness reductions during the passage of the metal strip through each stand of a tandem mill or during each pass through a reversing mill. Pass-schedules are normally designed such that the target thickness is obtained in a minimum number of passes (for reversing mills) or in the minimum amount of time and energy for tandem mills. In addition to rolling mill mechanical and electrical limitations, constraints related to the dimensional quality of the rolled strip generate limits to the amount of thickness reduction in any given pass. Since the roll-stack deflection, strip thickness profile, and corresponding strip flatness are functions of the applied rolling force, the dimensional quality of the rolled strip is influenced by the pass-schedule employed. It is desirable, therefore, to formulate and solve the on-line pass-schedule optimization problem for any type of rolling mill. This requires a rapid and accurate strip profile and flatness model, such as that which has been introduced and presented here.
Presented is a simple example of the presented model’s application to pass schedule optimization for a 20-High Sendzimir mill. As mentioned earlier, to realize minimal changes strip flatness during rolling, it is desirable to maintain a constant C25 crown ratio of the strip. The optimization example therefore adjusts the exit gages of an existing pass schedule to ensure that the desired strip crown ratio is achieved, given a degree of strip crown control. In the case of the 20-High Sendzimir mill, we elect to apply parabolic displacement of the upper bearing shafts in order to increase or decrease the strip crown ratio.

Table 9 – Pass schedule initial exit gage and unit force

<table>
<thead>
<tr>
<th>Pass No.</th>
<th>Exit Gage, h (mm)</th>
<th>Unit Force, p (N/mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.82</td>
<td>4893.48</td>
</tr>
<tr>
<td>2</td>
<td>3.09</td>
<td>5495.77</td>
</tr>
<tr>
<td>3</td>
<td>2.52</td>
<td>5566.50</td>
</tr>
</tbody>
</table>

Table 9 illustrates initial pass exit gage and unit rolling force for 3 passes of a sample pass schedule on a 20-High mill. We seek to optimize the exit gage of each pass to ensure desired C25 strip crown ratio for a specific level of crown control capability. The initial gage and strip width in this example are 5 mm and 1000 mm respectively. The material type is 304 stainless steel.
We formulate a constrained optimization problem to minimize relative perturbations $x_1$ and $x_2$ to the pass exit gages $h_1$ and $h_2$ respectively. Note that the exit gage of the final pass is fixed. The objective function, $f(x_1, x_2)$, to minimize is:

$$f(x_1, x_2) = x_1^2 + x_2^2$$  \hspace{1cm} (28)

As indicated in Equations 29-31, optimization constraints are imposed such that the C25 crown ratio can be achieved for each of the three passes, and also so that maximum unit rolling force is not exceeded.

$$-\frac{C_{25_{\text{max}},i}}{C_{25_T}} + 1 \leq 0$$ \hspace{1cm} (29)

$$\frac{C_{25_{\text{min}},i}}{C_{25_T}} - 1 \leq 0$$ \hspace{1cm} (30)

$$\frac{1}{p_{\text{max}}} \left( p_i + \frac{\partial p_i}{\partial h_i} x_i h_i + \frac{\partial p_i}{\partial H_i} x_{i-1} h_{i-1} \right) - 1 \leq 0$$ \hspace{1cm} (31)

In these equations, the index $i$ refers to the pass number (1 to 3). For each pass, Equations 29 and 30 incorporate respectively the maximum and minimum calculated strip crown ratios based on some specified bounds of the crown control mechanisms in the 20-High mill. Equation 31 uses sensitivities of unit rolling force with respect to entry and exit gages to ensure that that the new unit force, $p_i$, does exceed the maximum unit
force, \( p_{\text{max}} \). Software tools for solving such optimization problems are widely available. The challenge is to create accurate and efficient mathematical representations of the physical phenomena—which in this case is the strip crown.

Figures 28a and 28b illustrate the influence of the crown control on the force distribution and strip thickness profile respectively for pass 1 of the 20-High mill schedule in this example. Table 10 summarizes interesting results of the optimization example, the objective of which was to adjust the pass exit gages to ensure sufficient crown control authority for achieving a target crown of 0.30%. A close look at the results shows that it was necessary to decrease the exit gage on pass 1 to increase the nominal crown from 0.275% to 0.286% so that the crown control could achieve the 0.30% target. Pass 2 exit gage was then decreased by 3.25%, allowing pass 3 to achieve the 0.30% target crown with the available control and its reduced entry gage.

![Influence of Crown Control on Force Distribution](image)

**Figure 28a – Influence of crown control on rolling force distribution**
Table 10 – Pass schedule optimization for 20-High mill to enable target crown of 0.30%

<table>
<thead>
<tr>
<th>Pass No.</th>
<th>Final exit gage (mm)</th>
<th>Exit gage change (%)</th>
<th>Initial Crown (%)</th>
<th>Final Crown (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.78</td>
<td>-1.17</td>
<td>0.275</td>
<td>0.286</td>
</tr>
<tr>
<td>2</td>
<td>2.99</td>
<td>-3.25</td>
<td>0.363</td>
<td>0.394</td>
</tr>
<tr>
<td>3</td>
<td>2.52</td>
<td>N/A</td>
<td>0.442</td>
<td>0.409</td>
</tr>
</tbody>
</table>

Figure 28b – Influence of crown control on strip thickness profile
5.2 Roll Profile Optimization for a 4-High Temper Mill

The subject of roll profile optimization involves the determination of the optimum diameter profiles applied to individual rolls via mechanical grinding means. This approach is one method of counteracting the naturally occurring convex strip profile that results during rolling. Parabolic profiles are frequently ground onto work rolls and back-up rolls for this purpose. Since different rolling schedules usually require different magnitudes and forms of the ground profiles, a problem arises in finding the optimum profile to satisfy a diverse product mix, and yet preclude the necessity to change rolls frequently. In this regard, a linear programming problem is formulated for use with strip profile sensitivity calculations to optimize the ground profiles of work rolls and back-up rolls on a 4-high temper mill with a widely ranging product mix [53].

Optimization of the ground profiles of work rolls and back-up rolls (roll crowns) is critical to the productivity of many mills and to the quality of the rolled strip. Without suitable roll profiles, the rolling operation requires excessive roll inventory (with various roll crowns), greater on-line strip profile and flatness control authority, and frequent roll changes - leading to operating inefficiencies and higher operation costs. In addition, desired strip flatness may be difficult or impossible to achieve if roll ground profiles are not suitable for the products rolled.

In the past, theoretical methods have been applied to optimize the roll profiles for multiple-stand tandem type rolling mills [55-57]. However, these methods were not suited to a single stand mill with a much larger mix of strip widths, gauges, and material
grades. As a result, the most common method to assign ground roll profiles for a single stand mill relies on trial-and-error methods and operating experience. One approach to optimizing the roll crowns of a 4-high single stand mill is to apply a novel linear programming optimization technique in conjunction with strip profile sensitivity calculations. This technique is able to optimize the profiles of both the work rolls and back-up rolls for a varied product mix, based on the effects of various strip profile control devices of the mill. The linear constraint equations in the optimization problem are established using the natural strip profile (or crown) and the strip profile effects due to the various control devices. Additional linear constraint equations are developed using the strip profile criteria [58]. The optimal roll profiles are then solved using the conventional “simplex” linear programming technique [59].

A method to calculate strip crown is prerequisite in the formulation of the roll optimization problem presented here. Specifically, the sensitivities or partial derivatives of the total strip crown function with respect to its contributing factors are required to establish the matrix constraint equations. In the past, Fapiano published well-received work using crown sensitivities and the linear programming technique to perform an optimal crown and shape set up calculation of multi-stand mills [56]. The algorithm allowed him to include provisions for recommended roll crown changes in case the target strip crown could not be met at particular stands. His assumption was that the parabolic ground roll crown could be substituted for an equivalent amount of “crown” produced by roll bending mechanisms. The work presented here extends the linear programming technique in a direction that determines the most suitable ground roll crowns over the
the entire range of product mix. Depending on formulation, it can consider combinations of several components of ground roll crowns, including parabolic profiles, quartic profiles, tapers or chamfers. Although, as noted earlier, the method is particularly applicable to single-stand “jobbing” type mills, the concept may be applied to a hot or cold mill with any number of stands, given the strip profile sensitivity functions.

Guo and others have published equations to represent the total exit strip crown in a manner analogous to Equation 32 below, which considers a single stand only [60]. To identify the total exit strip crown at every stand in a multi-stand mill, Guo used cascade-effect matrices and vectors in lieu of the scalar terms of Equation 32.

\[
Cr = N + \alpha_E E + \sum_{i}^{K} \alpha_i X_i = Cr(N, E, X_i)
\]

where \( \alpha_E \) and \( \alpha_i \) are scale factors

Equation 32 states that the total exit strip crown, \( Cr \), is the sum of the various crown effects, which include the natural strip crown, \( N \), a contribution, \( \alpha_E E \), from the entry strip crown, and up to \( K \) other effects, \( X_i \), from items such as work roll bending, backup roll bending, roll shifting, roll thermal & wear, and the ground crown components of work rolls and backup rolls.

From the right side of Equation 32, an approximate differential expression for the change in total exit strip crown for one rolling condition can be made as follows:
\[ \Delta Cr = \frac{\partial Cr}{\partial N} \Delta N + \frac{\partial Cr}{\partial E} \Delta E + \sum_{i}^{K} \frac{\partial Cr}{\partial X_i} \Delta X_i \]  

Equation 33 can be restated to represent final values only instead of changes in the value of each variable if initial values are set to zero, where the subscript (2) represents the final state:

\[ Cr_{(2)} = \frac{\partial Cr}{\partial N} N_{(2)} + \frac{\partial Cr}{\partial E} E_{(2)} + \sum_{i}^{K} \frac{\partial Cr}{\partial X_i} X_{i(2)} \]  

In constructing the constraint equations, Equation 34 is employed to represent the target strip crown that is required for particular products considered. A question that may come to mind is how to determine the partial derivative functions for each product’s operating conditions. The preferred method to identify the partial derivatives (or crown sensitivities) is to use a strip profile crown model such as that introduced in this work in Section 3.2. Otherwise the partial derivatives can be determined by mill tests in conjunction with Equation 34 as one variable at a time is changed.

The generalized linear programming problem requires that a linear vector function, \( h = C^T x \), be maximized or minimized subject to a constraint matrix of the form \( Ax \leq b \) where \( x \geq 0 \). The optimized vector \( h \) represents a cost function relating the roll profile variables and other parameters. The constrained vector \( x \) represents any unknown variables that influence the exit strip crown. The function \( h \) is optimized using the simplex method of linear programming [59].
The simplex method is a popular technique used to find the optimum feasible vector (or optimum feasible solution); starting with a basic feasible vector, one proceeds successively to neighboring feasible vectors until an optimum solution is found such that \( h \) is maximized or minimized. Optimization using the linear programming method is preferable to other optimization methods whenever the domain of the vector \( x \) is restricted. This situation is particularly inherent in the optimization of roll crowns, since each rolled strip of a diverse and wide-ranging product mix imposes restrictions on the domain of possible roll crowns. Before formulating the constraint equations \( Ax \leq b \), Equation 34 can be expanded into the following form for one particular operating condition with specific material, width, entry gauge, and reduction:

\[
C_{r(2)} = \frac{\partial C_r}{\partial X_{RED(2)}} X_{RED(2)} + \frac{\partial C_r}{\partial E} E_{(2)} + \frac{\partial C_r}{\partial X_{WRB}} X_{WRB(2)} + \frac{\partial C_r}{\partial X_{BUB}} X_{BUB(2)} + \frac{\partial C_r}{\partial X_{WRC}} X_{WRC(2)} + \frac{\partial C_r}{\partial X_{BUC}} X_{BUC(2)} + \frac{\partial C_r}{\partial X_{BUT}} X_{BUT(2)} \]

(35)

where the independent variable at their final state are:

- \( X_{RED} \) = percent reduction
- \( X_{WRB} \) = work roll bending force
- \( X_{BUB} \) = backup roll bending force
- \( X_{WRC} \) = parabolic work roll crown
- \( X_{BUC} \) = parabolic backup roll crown
- \( X_{BUT} \) = backup roll taper (chamfer) gradient
In the above equation, for later convenience, the natural crown $N$ has been replaced by a percent reduction $X_{\text{RED}}$. For the case-study mill in this work, specific variables have been selected from the wide range of possible variables that signify active or passive crown control devices and that constitute terms inside the summation of Equation 34. The selection of individual terms here is arbitrary and is dictated by the control devices present on the 4-high temper mill. The back up roll taper is chosen based on the studies of natural strip profile behavior on 4-high mills. The taper is configured such that it is initiated at a distance of 75% of the roll barrel length with respect to the. The most suitable taper steepness is unknown and is part of the optimization problem. In addition to the backup roll taper, conventional parabolic crowns are selected as additional ground crown components on both the work rolls and backup rolls. Since the objective is to determine the optimum roll crown components for application over the entire product range, Equation 35 should be written for the target strip crown of those operating conditions that make up the product envelope boundaries. This means that the optimized roll crown components should satisfy operating requirements for coils that represent extremes in material strength, width, gauge, and reduction. For the temper mill considered here, a minimum target elongation (reduction) is required on all products, thereby exempting reduction as a factor in product envelope. Hence, if two extremes conditions are applied to each of the remaining three envelope criteria, a total of eight cases ($2^3$) will result, as shown in Table 11.
Table 11 – Product envelope considered

<table>
<thead>
<tr>
<th>Prod. Case No.</th>
<th>Strip Modulus</th>
<th>Strip Width</th>
<th>Entry Gauge</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Soft (3.3 Mpsi)</td>
<td>Narrow (46 in.)</td>
<td>Light (3/16 in.)</td>
</tr>
<tr>
<td>2</td>
<td>Soft (3.3 Mpsi)</td>
<td>Narrow (46 in.)</td>
<td>Heavy (1/2 in.)</td>
</tr>
<tr>
<td>3</td>
<td>Soft (3.3 Mpsi)</td>
<td>Wide (98 in.)</td>
<td>Light (3/16 in.)</td>
</tr>
<tr>
<td>4</td>
<td>Soft (3.3 Mpsi)</td>
<td>Wide (98 in.)</td>
<td>Heavy (1/2 in.)</td>
</tr>
<tr>
<td>5</td>
<td>Hard (8.5 Mpsi)</td>
<td>Narrow (46 in.)</td>
<td>Light (3/16 in.)</td>
</tr>
<tr>
<td>6</td>
<td>Hard (8.5 Mpsi)</td>
<td>Narrow (46 in.)</td>
<td>Heavy (1/2 in.)</td>
</tr>
<tr>
<td>7</td>
<td>Hard (8.5 Mpsi)</td>
<td>Wide (98 in.)</td>
<td>Light (3/16 in.)</td>
</tr>
<tr>
<td>8</td>
<td>Hard (8.5 Mpsi)</td>
<td>Wide (98 in.)</td>
<td>Heavy (1/2 in.)</td>
</tr>
</tbody>
</table>

Success in the determination of roll crown components is achieved if one can roll flat strip on all products in accordance with constant crown ratio requirements. If the product envelope is very small, the necessity of online control devices such as roll bending or roll shifting mechanisms becomes less important. On the other hand, sufficiently large ranges in online crown control tools are critical in obtaining flat strip for large product mix envelopes. As will be seen next, the “best” roll crown components are often those that can simply enable flat strip to be rolled within the limits of online control devices – regardless of the specific set-points of those devices. This means that the domain of the vector $x$ becomes the dominating factor in the optimization problem, rather than the form of the optimized function $h$. It may turn out that for a given product envelope, it is physically impossible to meet the crown and shape targets with a given specific crown control capability. The simplex optimizing routine delivers no solution in this case.
In order to establish the domain of the unknown variables sought here (work roll crown, backup roll crown, and backup roll taper gradient) the control range of each online device has to be taken into consideration. One can thus write two instances of Equation 35 – where the two instances represent extreme conditions in the application of the online crown control tools. For a specific target exit crown and known entry crown, the resulting equations define the domain of the roll crown components within which the target crown can be obtained. In the case of the 4-high temper mill studied here, the extreme conditions involve maximum and minimum work roll and backup roll bending. As to a specific mill, other types of online devices may be included, but some difficulty may arise in determining the partial derivatives at their extreme application points, particularly in the absence of a tuned offline crown model. For mills with work roll and backup roll bending, the two instances of Equation 35 can be written as follows, where the known quantities have been moved to the left hand side:

For maximum work roll bending and maximum back-up roll bending:

\[
(Cr_{(2)} - \frac{\partial Cr}{\partial X_{RED}} X_{RED_{(2)}} - \frac{\partial Cr}{\partial E} E_{(2)} - \frac{\partial Cr}{\partial X_{WRB}} X_{WRB_{(2), \text{MAX}}} - \frac{\partial Cr}{\partial X_{BUB}} X_{BUB_{(2), \text{MAX}}}) \]  

\[= \frac{\partial Cr}{\partial X_{WRC}} X_{WRC_{(2)}} + \frac{\partial Cr}{\partial X_{BUC}} X_{BUC_{(2)}} + \frac{\partial Cr}{\partial X_{BUT}} X_{BUT_{(2)}} \]

For minimum work roll bending and minimum back-up roll bending:
The equations above define a 3-dimensional domain of acceptable roll crown components, $X_{\text{WRC}}$, $X_{\text{BUC}}$, $X_{\text{BUT}}$, for one particular operating condition and set of product attributes. If the 3-dimensional domain space is reduced to two dimensions by removing the backup roll taper gradient component $X_{\text{BUT}}$, the reduced planar domain can be interpreted graphically. Figure 29 depicts a sample work roll - backup roll parabolic crown domain for one product only. At any point $(X_{\text{WRC}}, X_{\text{BUC}})$ inside the curves, the target crown can be met without the roll bending forces exceeding their limits. Adding more products, especially those that define the product mix envelope, further restricts the roll crown domain as illustrated in Figure 30. The very narrow domain of Figure 30 results when the roll crowns are required to satisfy wide-ranging products. Moreover, Figure 30 only represents the hardest material type in the product set of Table 11. When soft materials of equivalent widths and gauges are added, it is intuitive that no feasible domain exists. To address this typical problem, a method was devised that allows changes in the natural crown contribution to exit strip crown by optimizing the percent reduction within allowable limits. It so happens that for the case-study temper mill, although a minimum of 2% elongation is required, 3 or 4% is still acceptable in meeting all mechanical property requirements. At the same time, this approach facilitates improvements in product yield.
Figure 29 – Example domain of roll crown components for one product
(2D only, backup roll taper gradient excluded)

Figure 30 – 2D domain of roll crown components
(considering width and gauge envelope for hardest material type only)
Traditionally, the operating philosophy on temper mills has been to “roll for flatness,” while striving to meet the minimum target reduction requirements. The method presented allows temper mills to roll for optimum flatness (shape) and optimum reduction (elongation) at the same time. Hard limits on percent reduction can easily be built into the constraint matrix $A$. In Figures 29 and 30, increasing or decreasing the percent reduction (and subsequent natural strip crown) is analogous to shifting a pair of curves up or down respectively. In this manner, a common roll crown component domain can be sought for all products in the mix. In attempting to find a common roll crown domain, an additional variable representing a possible change in reduction is added to Equations 36 and 37, which now take the following form.

For maximum work roll and back-up roll bending but minimum roll crowns:

$$
(Cr_{(2)} - \frac{\partial Cr}{\partial X_{RED}} X_{RED(2)} - \frac{\partial Cr}{\partial E} E_{(2)} - \frac{\partial Cr}{\partial X_{WRB}} X_{WRB(2),\text{MAX}} - \frac{\partial Cr}{\partial X_{BUB}} X_{BUB(2),\text{MAX}})
\geq \frac{\partial Cr}{\partial X_{WRC}} X_{WRC(2)} + \frac{\partial Cr}{\partial X_{BUC}} X_{BUC(2)} + \frac{\partial Cr}{\partial X_{BUT}} X_{BUT(2)} + \frac{\partial Cr}{\partial X_{RED}} \delta_{REDi}
$$

(38)

For minimum work roll and back-up roll bending but maximum roll crowns:

$$
(Cr_{(2)} - \frac{\partial Cr}{\partial X_{RED}} X_{RED(2)} - \frac{\partial Cr}{\partial E} E_{(2)} - \frac{\partial Cr}{\partial X_{WRB}} X_{WRB(2),\text{MIN}} - \frac{\partial Cr}{\partial X_{BUB}} X_{BUB(2),\text{MIN}})
\leq \frac{\partial Cr}{\partial X_{WRC}} X_{WRC(2)} + \frac{\partial Cr}{\partial X_{BUC}} X_{BUC(2)} + \frac{\partial Cr}{\partial X_{BUT}} X_{BUT(2)} + \frac{\partial Cr}{\partial X_{RED}} \delta_{REDi}
$$

(39)
where $\delta_{REDi}$ indicates an increase in the percent reduction for the $i^{th}$ product in the constraint matrix $A$, which comprises $n$ unique product specifications. With the re-inclusion of $X_{BUT}$, the 2-dimensional subspace of Figures 29 and 30 now becomes $(n+3)$-dimensional, thereby offering the potential for many additional feasible solutions. Equations 38 and 39 are written as inequalities since it is clear that any set of roll crown components within the bounded domain can produce the target strip crown without exceeding the limitations of roll bending. The entire equation set can be written in simplified notation and in compliance with the inequality form of $Ax \leq b$:

$$- \alpha_{WRCi} \cdot X_{WRC} - \alpha_{BUCi} \cdot X_{BUC} - \alpha_{BUTi} \cdot X_{BUT} - \alpha_{REDi} \cdot \delta_{REDi} \leq -b_i$$  \hfill (40)$$

$$\alpha_{WRCi} \cdot X_{WRC} + \alpha_{BUCi} \cdot X_{BUC} + \alpha_{BUTi} \cdot X_{BUT} + \alpha_{REDi} \cdot \delta_{REDi} \leq c_i$$  \hfill (41)$$

$$X_A \leq X_{A,\text{MAX}}$$  \hfill (42)$$

$$- X_A \leq -X_{A,\text{MIN}}$$  \hfill (43)$$

where $i = 1, 2, \ldots, n$

and $X_A = X_{WRC}, X_{BUC}, X_{BUT}, \delta_{REDi}$

In Equations 40 and 41, $\alpha$ is used to signify $\frac{\partial Cr}{\partial x_A}$. Term $b_i$ is the left hand side of Equation 39 and $c_i$ is the left hand side of Equation 38 for the $i^{th}$ product criteria.
After establishing the linear programming constraint equation set, it can be solved using the simplex (or other) method, together with a cost function $h = C^T x$ of the user’s choice. The coefficient vector $C$ is generally used to impose weights on the components of the optimized vector $x$, in accordance with preferences on the unknown variables $X_{WRC}$, $X_{BUC}$, $X_{BUT}$, and $\delta_{RED}$. In this work, the variables $\delta_{RED}$, which represent additional elongation beyond 2%, are assigned a very high cost in $h$. This means that they would only assume non-zero in values when necessary to obtain a common feasible domain for the 3-dimensional roll crown components.

Tables 12, 13, and 14 show a sampling of results that were generated in the application of the linear programming method to the hot-band temper mill:

Table 12 – Strip Crown Partial Derivatives (based on Transport Matrix Method [37])

<table>
<thead>
<tr>
<th>Case No.</th>
<th>$\alpha_E$ (mil / mil)</th>
<th>$\alpha_{RED}$ (mil / %)</th>
<th>$\alpha_{WRB}$ (mil / ton)</th>
<th>$\alpha_{BUB}$ (mil / ton)</th>
<th>$\alpha_{WRC}$ (mil / mil)</th>
<th>$\alpha_{BUC}$ (mil / mil)</th>
<th>$\alpha_{BUT}$ (mil / [mil/in])</th>
</tr>
</thead>
<tbody>
<tr>
<td>From Table 1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.24538</td>
<td>1.40755</td>
<td>-0.01651</td>
<td>-0.00309</td>
<td>-0.13579</td>
<td>-0.06303</td>
<td>-0.81884</td>
</tr>
<tr>
<td>2</td>
<td>0.15693</td>
<td>2.54510</td>
<td>-0.01522</td>
<td>-0.00424</td>
<td>-0.18621</td>
<td>-0.08661</td>
<td>-0.71465</td>
</tr>
<tr>
<td>3</td>
<td>0.42560</td>
<td>2.09075</td>
<td>-0.04187</td>
<td>-0.00669</td>
<td>-0.31987</td>
<td>-0.13655</td>
<td>-1.45625</td>
</tr>
<tr>
<td>4</td>
<td>0.32535</td>
<td>4.56175</td>
<td>-0.06796</td>
<td>-0.01097</td>
<td>-0.52396</td>
<td>-0.22384</td>
<td>-2.41990</td>
</tr>
<tr>
<td>5</td>
<td>0.31224</td>
<td>1.85690</td>
<td>-0.00858</td>
<td>-0.00251</td>
<td>-0.10970</td>
<td>-0.05120</td>
<td>-0.41715</td>
</tr>
<tr>
<td>6</td>
<td>0.19484</td>
<td>3.24070</td>
<td>-0.01338</td>
<td>-0.00392</td>
<td>-0.17166</td>
<td>-0.08014</td>
<td>-0.65530</td>
</tr>
<tr>
<td>7</td>
<td>0.48521</td>
<td>2.52405</td>
<td>-0.02878</td>
<td>-0.00491</td>
<td>-0.23275</td>
<td>-0.10023</td>
<td>-1.05440</td>
</tr>
<tr>
<td>8</td>
<td>0.37580</td>
<td>5.47650</td>
<td>-0.05624</td>
<td>-0.00949</td>
<td>-0.45089</td>
<td>-0.19379</td>
<td>-2.07525</td>
</tr>
</tbody>
</table>
Table 13 – Random Constraints for Roll Crown Components and Reduction Modifiers

<table>
<thead>
<tr>
<th>Result</th>
<th>Constraint on Work Roll Crown (mil)</th>
<th>Constraint on Backup Roll Crown (mil)</th>
<th>Constraint on Backup Roll Taper (mil/in)</th>
<th>Constraints on Add’l Reduc. $\delta_{\text{RED}_i}$ (%)</th>
<th>Solution Found</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>$0 \leq X_{\text{WRC}} \leq 5$</td>
<td>$0 \leq X_{\text{BUC}} \leq 20$</td>
<td>$0 \leq X_{\text{BUT}} \leq 2$</td>
<td>$0 \leq \delta_{\text{RED}_i} \leq 0.5$</td>
<td>No</td>
</tr>
<tr>
<td>B</td>
<td>$0 \leq X_{\text{WRC}} \leq 5$</td>
<td>$0 \leq X_{\text{BUC}} \leq 20$</td>
<td>$0 \leq X_{\text{BUT}} \leq 2$</td>
<td>$0 \leq \delta_{\text{RED}_i} \leq 2.0$</td>
<td>Yes*</td>
</tr>
<tr>
<td>C</td>
<td>$0 \leq X_{\text{WRC}} \leq 5$</td>
<td>$0 \leq X_{\text{BUC}} \leq 20$</td>
<td>$0 \leq X_{\text{BUT}} \leq 5$</td>
<td>$0 \leq \delta_{\text{RED}_i} \leq 0.5$</td>
<td>No</td>
</tr>
<tr>
<td>D</td>
<td>$0 \leq X_{\text{WRC}} \leq 10$</td>
<td>$0 \leq X_{\text{BUC}} \leq 20$</td>
<td>$0 \leq X_{\text{BUT}} \leq 5$</td>
<td>$0 \leq \delta_{\text{RED}_i} \leq 0.5$</td>
<td>Yes*</td>
</tr>
<tr>
<td>E</td>
<td>$0 \leq X_{\text{WRC}} \leq 10$</td>
<td>$0 \leq X_{\text{BUC}} \leq 10$</td>
<td>$0 \leq X_{\text{BUT}} \leq 5$</td>
<td>$0 \leq \delta_{\text{RED}_i} \leq 2.0$</td>
<td>Yes*</td>
</tr>
<tr>
<td>F</td>
<td>$0 \leq X_{\text{WRC}} \leq 5$</td>
<td>$0 \leq X_{\text{BUC}} \leq 5$</td>
<td>$0 \leq X_{\text{BUT}} \leq 5$</td>
<td>$0 \leq \delta_{\text{RED}_i} \leq 2.0$</td>
<td>No</td>
</tr>
<tr>
<td>G</td>
<td>$0 \leq X_{\text{WRC}} \leq 0$</td>
<td>$0 \leq X_{\text{BUC}} \leq 30$</td>
<td>$0 \leq X_{\text{BUT}} \leq 10$</td>
<td>$0 \leq \delta_{\text{RED}_i} \leq 2.0$</td>
<td>Yes*</td>
</tr>
<tr>
<td>H</td>
<td>$0 \leq X_{\text{WRC}} \leq 20$</td>
<td>$0 \leq X_{\text{BUC}} \leq 0$</td>
<td>$0 \leq X_{\text{BUT}} \leq 10$</td>
<td>$0 \leq \delta_{\text{RED}_i} \leq 0.0$</td>
<td>No</td>
</tr>
<tr>
<td>I</td>
<td>$0 \leq X_{\text{WRC}} \leq 20$</td>
<td>$0 \leq X_{\text{BUC}} \leq 0$</td>
<td>$0 \leq X_{\text{BUT}} \leq 10$</td>
<td>$0 \leq \delta_{\text{RED}_i} \leq 1.0$</td>
<td>Yes*</td>
</tr>
</tbody>
</table>

*Product no. 6 (hard material, narrow width, heavy gauge) could not meet minimum 2% reduction (1.6% at best)

Table 14 – Roll Crown Optimization Results

<table>
<thead>
<tr>
<th>Result</th>
<th>Work Roll Crown (mil)</th>
<th>Backup Roll Crown (mil)</th>
<th>Backup Roll Taper (mil/in)</th>
<th>Add’l Reduction $\delta_{\text{RED}_i}$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td>5.00</td>
<td>20.00</td>
<td>1.783</td>
<td>$\delta_5=0.42, \delta_6=1.31, \delta_7=0.50$</td>
</tr>
<tr>
<td>D</td>
<td>10.00</td>
<td>14.85</td>
<td>0.000</td>
<td>$\delta_5=0.50$</td>
</tr>
<tr>
<td>E</td>
<td>0.00</td>
<td>10.00</td>
<td>3.840</td>
<td>$\delta_5=0.68, \delta_6=1.33, \delta_7=0.53$</td>
</tr>
<tr>
<td>G</td>
<td>0.00</td>
<td>30.00</td>
<td>0.995</td>
<td>$\delta_5=0.65$</td>
</tr>
<tr>
<td>I</td>
<td>19.61</td>
<td>0.00</td>
<td>0.000</td>
<td>$\delta_5=1.0, \delta_7=0.25$</td>
</tr>
</tbody>
</table>

Examination of Tables 12, 13, and 14 illustrates some general results of applying the linear programming method to the 4-high hot-band temper mill. From Table 12 it is evident that the wide, heavy-gauge materials (products 4 and 8) are most sensitive to the influences upon strip crown. The partial derivatives in this table are functions of mill configuration, geometry, operating conditions, and product attributes. Different mills and product types will produce sensitivity values different from those shown. In Table 13
some arbitrary upper and lower bounds were applied to the work roll crown, backup roll crown, backup roll taper gradient, and percent reduction modifiers. Based on these constraints and the constraints provided by the 2n product Equations 40 and 41, the solution result was obtained using the simplex method. As noted by the asterisk, in no case could a solution be found that enabled product no. 6 (hard material, narrow width, heavy gauge) to meet the minimum 2% reduction criterion. Instead, in every case it never reached more than 1.6%. If one were to allow much larger increases in the percent reduction of the soft materials, or allow very large roll crowns, it might be possible to find a common solution. However, for mechanical property requirements of the coils, the first suggestion is not sound. The latter suggestion is also undesirable since it may lead to roll loose-edge contact problems when the mill is not heavily loaded. A second look at Figure 30 sheds some light on why product no. 6 presents difficulties. By requiring the largest amount of total roll crown, product no. 6 is responsible for the lower boundary of roll crown domains in the 2-D graphical representation. Since the softer materials produce less natural crown due to lower rolling forces, it is difficult to match the domains of both the soft and hard materials without very large increases in reduction (to require larger roll crowns) of the soft materials. After commissioning optimized rolls (similar to those of result E) the case-study mill did have difficulties reaching the reduction target for product no. 6, although the actual percent reduction to obtain flat strip was slightly lower than anticipated. Except for the wide, soft, light gauge material, all products (including those well within the envelope and not sampled) were able to meet strip flatness and reduction requirements.
5.3 Reliability Analysis of Strip Profile for a 4-High Mill

This example presents an application of the Hasofer-Lind reliability analysis method to study the manufacturing performance of the strip thickness profile and corresponding flatness [52]. Consideration is given to the uncertain nature of the primary variables that influence the resulting strip thickness profile during rolling. For a selected operating condition and a particular rolling mill, two performance functions are generated that represent limit-states (extremes) of the strip thickness profile that allow for suitable flatness of the rolled strip and acceptable material yield loss. The limit-states are derived to represent the maximum and minimum allowable relative deviations from a rectangular strip profile. The random parameters in the performance functions are modeled as normally distributed independent variables, and include the strip compressive yield stress, work roll elastic modulus, work roll crowns, and strip entry crown. Using the Hasofer-Lind iterative scheme, a reliability statistic is obtained for the resulting strip profile. The calculated reliability index provides an estimate of the performance reliability, which in turn provides valuable insight into the quality of the rolling process and identifies which uncertain variables need to be addressed in order to improve the rolling quality.

Accurate prediction of the strip profile as presented in this work is important in improving the quality of the rolled strip. Even with an accurate mathematical model, however, the calculation may be complicated by the uncertain (random) nature of several variables in the rolling process. For example, the nominal rolling force, which is needed to predict the strip profile, may be randomly affected by the uncertain nature of other key parameters. One major factor affecting the rolling force, for example, is the constrained
compressive yield strength of the material, which itself frequently varies, since it depends on many other process parameters like temperature, prior mechanical working, chemical composition, and strain history if the material is not fully annealed before subsequent rolling. The friction coefficient at the area of contact between the work rolls and the strip is another factor that affects the rolling force, and it depends on variable parameters such as the condition of the rolls, the rolling temperature, and the surface condition of the incoming strip. Since the work rolls are usually supplied by more than one manufacturer and are regularly changed, it may be difficult to precisely identify the elastic modulus and subsequent roll flattening that occurs. The roll flattening affects the contact area at the strip and is another factor that influences the magnitude of rolling force.

In addition to the difficulty of accurately predicting the actual rolling force, a challenge is often presented in identifying the amount of crown possessed by the strip as it enters the rolling mill. Since the strip may have been previously rolled on another mill, the effects of prior rolling may often only be estimated unless actual measurement of the entry crown is made. As mentioned in Section 1.3, as a countermeasure against excessively large strip crowns during rolling, specialized diameter profiles are frequently ground onto both the work rolls and back-up rolls in order to modify the contact force distribution between the work rolls and the strip. These ground roll profiles (or roll crowns) are frequently parabolic, but may include tapering of the roll ends or more complex profiles such as “Continuously Variable Crown” (CVC) profiles, for example. In the absence of roll grinding machines with CNC (Computer Numerical Control)
capability, the ground roll profiles may vary randomly from design specifications because of the influences of the human grinding machine operators.

The random parameters chosen to illustrate an example of calculating the probability of achieving a desirable strip profile include the compressive yield strength of the strip, the elastic modulus of the work rolls, the strip crown entering the mill, and the ground diameter profile of the work rolls (roll crown). Intuitively, one might infer that these four random variables are independent, since their evolutionary processes are entirely unrelated. For this reason, and because no information suggesting any correlation can be found, they are considered as independent random variables. Although the friction coefficient between the strip and the work rolls was identified earlier as an uncertain contributor to the rolling force, it is not included in this example. The dependent random variable is the resulting crown of the strip after it exits the mill. Statistics for the mean and variance of the estimated random input variables are shown in Table 15.

<table>
<thead>
<tr>
<th>Random Variable</th>
<th>Symbol</th>
<th>Distribution / Type</th>
<th>Distribution Mean</th>
<th>Distribution Variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Entry Strip Crown (mils)</td>
<td>(X_1)</td>
<td>Normal / Independent</td>
<td>0.00</td>
<td>4.9E-3</td>
</tr>
<tr>
<td>Work Roll Parabolic Crown (mils)</td>
<td>(X_2)</td>
<td>Normal / Independent</td>
<td>10.00</td>
<td>9.0E-2</td>
</tr>
<tr>
<td>Strip Compressive Stress (Mpsi)</td>
<td>(X_3)</td>
<td>Normal / Independent</td>
<td>0.159</td>
<td>2.5E-3</td>
</tr>
<tr>
<td>Work Roll Elastic Modulus (Mpsi)</td>
<td>(X_4)</td>
<td>Normal / Independent</td>
<td>30.00</td>
<td>2.5E-1</td>
</tr>
</tbody>
</table>

The performance functions used to calculate a reliability statistic are developed by identifying the range of acceptable strip exit crowns that provides for both suitable strip
flatness and acceptable material yield loss. In order for the rolled strip to meet manufacturing quality objectives, two important characteristics directly related to the strip exit crown are observed. First, the rolled strip should meet the flatness (shape) requirements. As discussed earlier in Section 1.2, strip exit crowns that are too large can induce excessively wavy edges on the strip after rolling. Conversely, the opposite type of flatness problem can occur if the crown ratio possessed by the strip exiting the mill is less than the crown ratio of the strip upon entering the mill. The latter strip flatness problem is characterized by “buckles” in the center of the strip periodically spaced along its length. The center-buckle and wavy-edge flatness problems, among others, were illustrated previously in Figure 6. Rolling engineers are challenged to identify the range of acceptable strip exit crown values that will cause neither unacceptable wavy-edge nor unacceptable center-buckle conditions. In most rolling operations, and particularly in hot rolling where the ratio of strip thickness to width is large, the crown ratio can be increased or decreased a small amount before a noticeable change in the flatness is observed. The crown ratio range that does not affect the existing strip flatness is known as the flatness “dead-band.” or shape “dead-band.” Outside the dead-band, strip flatness may change as some function of the crown ratio change, as illustrated in Figure 31. The dead-band for a wavy-edge condition may be different from that of a center-buckle condition, as depicted by the quantities $B_1$ and $B_2$ respectively in Figure 31.

The second strip profile characteristic important for meeting manufacturing quality objectives relates to material yield loss. Since only the C25 edge thickness of the strip is typically verified by end users, the magnitude of the C25 strip crown should not
be so great as to promote yield losses. This requirement imposes a second upper limit on the strip exit crown ratio.

Based on the preceding discussion, upper and lower limits on the strip exit crown ratios can now be imposed to generate limit-state functions for the reliability analysis. In particular, a lower C25 crown ratio limit, $CR_{\text{min}}$, is imposed to avoid center-buckle flatness problems, while an upper C25 crown ratio limit, $CR_{\text{max}}$, is imposed to prevent wavy-edge flatness problems and excessive yield loss. Note that $CR_{\text{max}}$ is actually the smaller of the two maximum crown ratios representing thresholds for wavy-edge and yield-loss limits, respectively. As a result, two performance functions can be written as:

**Figure 31 – Strip Flatness change as a function of crown ratio change**
In Equations 44a and 44b, \( CR \) is simply the predicted strip crown ratio calculated using the C25 crown, as discussed in Section 1.2. A positive value for both performance functions indicates that the C25 crown ratio is in the “safe” random variable domain for the reliability problem posed. If either performance function is negative, the strip exit crown ratio is in the “failed” random variable domain, due to corresponding limit imposed by \( CR_{\text{max}} \) or \( CR_{\text{min}} \). The performance functions introduced above, together with the distribution information of the random variables in Table 15, can be used to perform a reliability analysis of the strip exit crown.

Since the performance functions specified by Equations 44a and 44b can be evaluated using the method to calculate the strip crown introduced in this work, and the random variables, \( X_1, X_2, \ldots, X_4 \), are considered independent and normally distributed, the reliability problem can be readily solved either by an analytical or a numerical procedure such as the First Order Reliability Method (FORM) outlined by Grandhi [52]. Grandhi’s method is one of several derivatives of the Hasofer-Lind iterative numerical solution procedure. From a given starting vector of the random variables, the Hasofer-Lind algorithm uses first-order gradient information of the limit-state function to iteratively search for a new design vector of the random variables \( X_1, X_2, \ldots, X_4 \), at which the limit-state function is closest to zero and the normalized design vector is minimized. The reliability index, \( \beta_{\text{HL}} \), corresponds to length of the normalized design vector.
Table 16 illustrates the results of a MATLAB program which was used to solve for the reliability index of the rolled strip crown ratio. The mean values of each of the random variables were used in the starting design vector, and a convergence criterion of 0.001% for the reliability index was assigned. As shown, the Hasofer-Lind algorithm converged from starting design vector with only two additional iterations. In Table 16, $g_1$ and $g_2$ represent the values of the performance functions specified by Equations 44a and 44b for each iteration. Since the initial values of $g_1$ and $g_2$ are positive, it is clear that the design vector with mean values of the random variable resides in the “safe” region. In other words, neither wavy-edge nor center-buckle would be expected.

Table 16 – Computation results for strip crown reliability analysis

<table>
<thead>
<tr>
<th>Iter.</th>
<th>$g_1$</th>
<th>$g_2$</th>
<th>$\beta_{HL1}$</th>
<th>$\beta_{HL2}$</th>
<th>$\alpha_{11}$</th>
<th>$\alpha_{12}$</th>
<th>$\alpha_{13}$</th>
<th>$\alpha_{14}$</th>
<th>$\alpha_{21}$</th>
<th>$\alpha_{22}$</th>
<th>$\alpha_{23}$</th>
<th>$\alpha_{24}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.9465</td>
<td>0.5535</td>
<td>1.3273</td>
<td>0.7762</td>
<td>-3.197</td>
<td>-2.229</td>
<td>0.8798</td>
<td>-0.2415</td>
<td>-0.259</td>
<td>-0.3197</td>
<td>-0.8798</td>
<td>-0.2415</td>
</tr>
<tr>
<td>2</td>
<td>0.0181</td>
<td>-0.0062</td>
<td>1.3534</td>
<td>0.7677</td>
<td>-3.334</td>
<td>-2.660</td>
<td>0.8691</td>
<td>-0.2519</td>
<td>-0.2534</td>
<td>-0.3127</td>
<td>-0.8854</td>
<td>-0.2387</td>
</tr>
<tr>
<td>3</td>
<td>5.72e-7</td>
<td>-9.29e-8</td>
<td>1.3534</td>
<td>0.7677</td>
<td>-3.334</td>
<td>-2.661</td>
<td>0.8690</td>
<td>-0.2521</td>
<td>-0.2534</td>
<td>-0.3127</td>
<td>-0.8854</td>
<td>-0.2388</td>
</tr>
</tbody>
</table>

The respective reliability indexes, $\beta_{HL1}$ and $\beta_{HL2}$, are seen to converge to values of 1.3534 and 0.7677. Using a cumulative density distribution table for standard normal random variables, estimates of the reliabilities of $g_1$ and $g_2$ are obtained as 91.1% and 77.9%, respectively [49]. This suggests that a failure of the strip crown ratio would most likely result in a center-buckle strip flatness condition. Furthermore, the overall system reliability based on the probability theory of Equation 45 is 69%, which represents the probability of not realizing wavy-edge and not realizing center-buckling condition. This
statistic is obtained as follows: if event “A” represents failure due to wavy-edge condition, and event “B” represents failure due to center-buckling, then using probability theory and the fact that events A and B are mutually exclusive, the overall system reliability for achieving acceptable flatness and acceptable yield loss is:

\[
P(\overline{A \cap B}) = P(\overline{A \cup B}) = 1 - P(A \cup B) = 1 - [P(A) + P(B)]
\]

\[
= 1 - P(A \cup B) = 1 - [P(A) + P(B)]
\]

\[
= 1 - (0.089 + 0.221)
\]

\[
= 0.69 \text{ or } 69\%
\]

where:

\[
P(A) = \text{Probability of event } A, \text{ or the probability of wavy-edge } = 1 - 0.911 = 0.089
\]

\[
P(B) = \text{Probability of event } B, \text{ or the probability of center-buckle } = 1 - 0.779 = 0.221
\]

and

\[
P(\overline{X}) = \text{Complement probability of event } X, \text{ or the probability of not achieving event } X
\]

To gain insight into which random variables are most influential in the reliability analysis, one can inspect the values of each \( \alpha \) in Table 16, which represent sensitivities (or direction cosines) of the reliability indexes. It can be seen that, for the given random variable statistics, \( \alpha_{13} \) and \( \alpha_{23} \) influence the results most. These are sensitivities of the reliability index with respect to the compressive yield stress random variable \( X_3 \). The remaining sensitivity parameters, representing the strip entry crown, work roll crown, and roll elastic modulus, are not significantly dominant over one another. It is noteworthy that the sensitivities for a given variable of \( g_1 \) and \( g_2 \) almost represent negatives of one
another, implying that it may be difficult to improve the reliability of wavy-edge conditions without increasing the likelihood of failure with regard to center-buckling. This means that either a reduction of the randomness in the uncertain variables or an implementation of adequate crown and flatness control systems are necessary in order to improve the reliability in this example.
6. SUMMARY

Presented has been the development of a new method to accurately and rapidly predict the strip thickness profile and corresponding flatness for any type of rolling mill, including both conventional vertical stand mills and cluster-type rolling mills. The new method combines the advantages of the Finite Element Method with solutions from classical solid mechanics to obtain a compact and flexible stiffness-based linear model that is straightforward to implement using conventional FEA algorithms. The presented model addresses the shortcomings of the conventional models; it is accurate, rapid, and flexible enough for application with typical on-line and off-line strip profile and flatness control systems for complex rolling mill configurations such as the 20-High Sendzimir mill. On-line applications for the presented model include pass-schedule optimization and the determination of transfer functions to compute profile or flatness control error signals. Off-line applications include roll profile optimization, rolling mill design, and the design of profile or flatness control hardware mechanisms. Examples have been provided for pass schedule optimization on a 20-High mill, roll profile optimization for a 4-High mill with both work roll bending and back-up roll bending. In addition, to introduce the important subject of Reliability Analysis to the metals industry, an example of using the developed model to estimate the probability of achieving desirable strip flatness with random input parameters was shown.
Validity and compactness of the presented method was investigated by comparing the results and solution times for the static deflection and strip profile in a 4-High mill with those obtained using the large-scale commercial Finite Element Analysis package ABAQUS. The deflection results of the developed model indicate close agreement with the ABAQUS model, yet the solution times and model memory-storage requirements remarkably favor the use of the developed model.

Since the results of only a single rolling mill were verified with large-scale FEA, as part of the validation process, a critical component of the developed model—the elastic foundation moduli between rolls in lengthwise contact—were studied for various roll geometries, loading condition, and frictional interactions. This enabled enhancement of load-deflection characteristics for the classic two-dimensional Hertz problem of solid cylinders in contact, and verified the significance of certain parameters relevant to the elastic foundation moduli. In particular, the elastic foundation parameters studied included the length to diameter ratios of the rolls, the friction coefficient between them, the magnitude of distributed contact load, and the relative position along the roll axes.

To expedite the efficiency of studying the elastic foundation moduli between contacting rolls, and identifying significant factors, an orthogonal Central Composite Design (CCD) analysis from the methods of Design of Experiments (DOE) was executed using several ABAQUS FEA simulations. Linear regression of the deflection results led to response surface equations for the ratio of the elastic foundation moduli at any axial position for a roll relative to the foundation moduli at the corresponding roll axis midpoint, which most represents the plane-strain assumption in the classic load-deflection solutions.
Validation of the model enables it to serve as the basic simulation tool for important rolling mill operational tasks such as those related to strip profile and flatness actuator optimization, pass schedule optimization, and optimization of mechanically ground roll profiles. Since uncertainties in the rolling process are abundant, the incorporation of the Reliability Analysis method may encourage metals manufacturers to adopt similar approaches to improve product quality and reduce product rejection rates. Furthermore, insight into the bottlenecks to improving the rolling process may be identified more easily.

**Future Directions**

Although the subject of dynamic analysis was introduced in Section 3.12, this work focused primarily on the theoretical development and validation of the static mill deflection model. Since vibration problems occur frequently in rolling, and such problems cause, at the very least, expensive surface quality problems with the rolled metal strip in the form of “chatter-marks,” a logical next step is to perform studies to calculate the mode shape and natural frequencies of vibration by solution of the readily-obtained eigenvalue problem. It may be interesting to examine the effect of different rolled metal products on the vibration characteristics.

Other important aspects of the developed model include the convergence behavior of the elastic foundation moduli with respect to intermediate contact-force calculations, and convergence similarities or discrepancies in the solution based on incremental
loading conditions as opposed to single-increment full loading of the mill. This is especially interesting with respect to “hard” nonlinear contact condition changes in which gaps open or close between the rolls under varying roll profiles and loading conditions.
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