A Study on the Effects of Coil Wedge During Rewinding of Thin Gauge Metals

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By

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ABSTRACT

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With the increase in demand for high quality thin gauge metals improvements to control systems that monitor strip characteristics must first be acquired. During the rewinding of sheet metal, little insight is known as to what stresses are attributed to the actual winding process, not simply those induced by the mill. When winding effects become too severe they will alter control system readings, prompting the need for a rapid model that can predict winding effects and filter them from current control systems. This research develops a new method to determine a 4th order Airy function that predicts the 2D stress state of the strip and allows for the filtering of winding effects efficiently and accurately. By using curve fits and function approximations, a polynomial equation can be used to predict the stress field without large scale finite elements or cumbersome Fourier series solutions. The new method is compared to finite element analysis results as well as industry data.
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### NOMENCLATURE

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N, i$</td>
<td>Number of wraps (or laps)</td>
</tr>
<tr>
<td>$j$</td>
<td>Widthwise segment</td>
</tr>
<tr>
<td>$D$</td>
<td>Diameter of mandrel</td>
</tr>
<tr>
<td>$D_1^N$</td>
<td>Diameter of winding at thicker edge of strip</td>
</tr>
<tr>
<td>$D_2^N$</td>
<td>Diameter of winding at thinner edge of strip</td>
</tr>
<tr>
<td>$T$</td>
<td>Thickness of strip on thick edge</td>
</tr>
<tr>
<td>$t$</td>
<td>Thickness of strip on thin edge</td>
</tr>
<tr>
<td>$\bar{t}$</td>
<td>Average thickness of strip</td>
</tr>
<tr>
<td>$a_j$</td>
<td>Elemental cross-sectional area across width of strip</td>
</tr>
<tr>
<td>$u$</td>
<td>Radial deflection of coil</td>
</tr>
<tr>
<td>$\bar{r}$</td>
<td>One time approximation of average radius</td>
</tr>
<tr>
<td>$r_{avg}$</td>
<td>Iterated average radius to satisfy applied tension criteria</td>
</tr>
<tr>
<td>$r_{(i+1,j)}$</td>
<td>Radial value across width of coil</td>
</tr>
<tr>
<td>$p_o$</td>
<td>External pressure</td>
</tr>
<tr>
<td>$p_i$</td>
<td>Internal pressure</td>
</tr>
<tr>
<td>$\delta(y)$</td>
<td>Deflection of strip as a function of $(y)$ position</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Maximum amount of deflection in the strip applied at $x = L_{MM}$</td>
</tr>
<tr>
<td>$\delta_{avg}$</td>
<td>Average deflection across the strip</td>
</tr>
<tr>
<td>$\delta^*$</td>
<td>Normalized deflection ratio</td>
</tr>
<tr>
<td>$\delta_{i}$</td>
<td>Deflection of $L_{(i+1)}$</td>
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<td>Deflection of $L_{MM}$</td>
</tr>
<tr>
<td>$W$</td>
<td>Width of strip</td>
</tr>
<tr>
<td>$W_C$</td>
<td>Characteristic width</td>
</tr>
<tr>
<td>$L_{MM}$</td>
<td>Length of strip from mill to mandrel</td>
</tr>
<tr>
<td>$L_{MM}$</td>
<td>Normalized length of planar region</td>
</tr>
<tr>
<td>$L_C$</td>
<td>Characteristic length</td>
</tr>
<tr>
<td>$C_{(i+1)}$</td>
<td>Circumference of outermost wrap</td>
</tr>
<tr>
<td>$L_{(i+1)}$</td>
<td>Total length of strip, $L_{MM} + C_{(i+1)}$</td>
</tr>
<tr>
<td>$T_{nom}$</td>
<td>Nominal tension applied to the strip by winder</td>
</tr>
<tr>
<td>$AR$</td>
<td>Aspect ratio of width and $L_{MM}$</td>
</tr>
<tr>
<td>$IR$</td>
<td>Inflection ratio</td>
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<tr>
<td>$\Phi$</td>
<td>Airy function</td>
</tr>
<tr>
<td>$E$</td>
<td>Modulus of Elasticity of strip</td>
</tr>
<tr>
<td>$G$</td>
<td>Modulus of Rigidity of strip</td>
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<tr>
<td>$\varepsilon_l$</td>
<td>Normal strain in the rolling direction of strip with length $L_{(i+1)}$</td>
</tr>
<tr>
<td>$\varepsilon_x$</td>
<td>Normal strain in the rolling direction of only $L_{MM}$ portion of strip</td>
</tr>
<tr>
<td>$\varepsilon_y$</td>
<td>Normal strain in the widthwise direction</td>
</tr>
<tr>
<td>$\gamma_{xy}$</td>
<td>In-plane shear strain</td>
</tr>
<tr>
<td>$\sigma_\theta$</td>
<td>Tangential stress</td>
</tr>
<tr>
<td>$\sigma_l$</td>
<td>Normal stress in rolling direction of strip with length $L_{(i+1)}$</td>
</tr>
<tr>
<td>$\sigma_x$</td>
<td>Normal stress in rolling direction of only $L_{MM}$ portion of strip</td>
</tr>
<tr>
<td>$\sigma_y$</td>
<td>Normal stress in widthwise direction</td>
</tr>
<tr>
<td>$\sigma_z$</td>
<td>Normal stress in out-of-plane direction</td>
</tr>
<tr>
<td>$\tau_{xy}$</td>
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1.0 INTRODUCTION

As products derived from metal sheet become smaller and more precise, the demand for higher quality raw materials also increases. When cold rolling thin metal strip or sheet containing asymmetric thickness profile, quality problems such as poor final flatness, surface imperfections, and strip defects are difficult to address. This is particularly true in the case of rolling and winding a strip with an asymmetric (or wedge) thickness profile. A wedge profile can lead to localised buckling (strain defects) due to high in-plane stress gradients or over-correction of the mill leading to edge buckle or tearing. The resulting poor strip flatness also decreases rolling productivity, and may render the products unusable in downstream manufacturing processes such as annealing, slitting, or stamping. An overview of the cold rolling and subsequent winding process has been discussed by Edwards and Boulton [1] as well as Roberts [2]. The focus of this work is to develop a numerical method to rapidly assess in-plane stress (or strain) and related flatness for strips with asymmetric thickness profile during the rolling/winding processes.

1.1 Cold Rolling Mills

The general idea of cold rolling is simple; apply a large enough pressure to a strip of metal in order to reduce its thickness (or gauge). Applying large magnitudes of
pressure normal to the surface of the strip will cause it to plastically deform. This plastic deformation will cause the strip to both decrease in thickness as well as elongate in length. This is due to plastic deformation being incompressible and the percent reduction in thickness is equal to the percent elongation in the lengthwise direction.

In order to maintain some level of control and increase the flatness of the strip, tension is applied to either side of the strip when entering and exiting the mill. Tension is applied by two electric motors that are used to wind and unwind the strip from mandrels on either side of the mill. This forward and backward tensioning reduces the amount of force required to plastically deform the strip and reduces the amount of energy consumed by the mill during manufacturing. Tension also increases the level of controllability during operation. The operator can increase tension on either side of the mill in order to aide in achieving a desired shape, finish, or strength characteristics in the finished product. Figure 1 shown below is an example of a 4 High Rolling Mill and two winders (coilers). 4 High simply means that there are a total of 4 rolls that make up the mill. Figure 1 shows two rolls on top and bottom of the strip with the larger rolls being backup rolls while those that contact the strip are called work rolls.
There are many types of cold rolling mills, each satisfying a specific requirement or characteristic pertaining to certain materials and/or finishes. Typically, 4 and 6 High mills are used for initial working of low to medium strength metals and have little flatness control capabilities. 12 and 20 High cluster-type mills however, are used in finishing high strength metals with very thin exit gauges and have more sophisticated control systems that allow for more complex settings on the mill to be used when monitoring the exit profile. Figure 2 and Figure 3 illustrate multiple types of mill configurations.

Figure 2: Mill configurations used for initial working

Figure 3: Mill configurations used for finishing and high strength metals
1.2 Mill Commands and Monitoring Capabilities

As alluded to previously, each mill has its own set of flatness control capabilities and monitoring tools. The mills typically used for initial working tend not to have a large amount of adjustment settings or sophisticated measurement systems. A typical 4 High mill will have 3 potential mill adjustments that can be made to the work rolls by the operator in order to combat flatness issues. Work roll bending occurs when a force is applied to the ends of the rolls, causing them to bend upward or downward depending on the direction of the applied load. Work roll shifting is accomplished by laterally moving the rolls in order to create a desired pressure distribution across the strip. Roll tilting is simply applying a larger amount of force on one side than the other. Figure 4 shows each command.

![Roll Bending](image1)
![Roll Shifting](image2)
![Roll Tilting](image3)

Figure 4: Mill commands for flatness control

The measuring devices used on each mill also differ greatly. Some mills use lasers, segmented sensor rolls, or a single gauge measurement device that measures exit gauge down the centre of the strip. This work deals with systems that involve segmented sensor rolls or shapemeter. A segmented sensor roll is made up of a series of bearings
that have force sensors mounted inside. This allows the strip to pass over the sensor roll and the sensors measure the magnitude of radial force the strip is applying to the bearings. If the strip is tighter in one region, the sensor will measure a higher magnitude of radial force. Figure 5 illustrates a sensor roll used with a 20 High mill. Image courtesy of Tenova I2S.

Figure 5: 20 High mill with segmented sensor roll (shapemeter)

Figure 5 depicts a strip in which the centre is relatively longer than the edges. Tighter regions (edges) obtain a higher tensile stress than the centre. This causes nonlinear stress behavior across the width of the strip and is recorded as a parabolic radial force distribution across the shapemeter. The control system on the mill then modifies the mill settings to combat this shape issue and attempts to decrease the nonlinear distribution as much as possible.
Making mill adjustments with incorrect or false readings can lead to mill over-correction. This may cause the strip to obtain regions containing large amounts of strain, creating buckles or wavy patterns in the strip. Figure 6 shows some common buckling forms during the rolling process.

![Common modes of strip defects](image)

**Figure 6: Common modes of strip defects**

### 1.3 Winding Effects

As shown in Figure 5, the stress across the width of the strip is directly correlated to the magnitude of the radial force distribution. The issue studied in this work pertains to the stress contribution caused by winding the strip unto the mandrel. When winding a non-ideal strip on top of itself many times, the difference in radial locations across the strip at the mandrel causes a winding stress gradient to occur. This winding stress interferes with the measurement taken at the shapemeter by altering the stress profile in the strip. The profile that is studied in this research is called wedge. Wedge simply means that the strip is thicker on one side than the other. Wedge is typically obtained by slitting a wider strip in half. Figure 7 depicts the full width and wedge profile obtained from slitting.
Figure 7: Full strip (top) and wedge after slitting (bottom)

As the wedge profile is wound upon itself many times, the stresses created by the winding are said to be fictitious in that once the strip is unwound, only deformation stresses imposed by the mill remain in the strip. This high fictitious stress gradient caused by winding a wedge profile can lead to over correction of the mill. As the winding stress on one side increases and the measurement records a large amount of force on one side of the strip, the control system adjusts the mill by increasing the amount of force on the side with a high stress gradient as to reduce the exit gauge and decrease the stress in the localized region. Figure 8 shows an example of initial wedge measurement and the control system feedback loop on a 4 High mill. Note that as the number of wraps increases, the measured flatness reading becomes more drastic and causes the control system to adjust the mill using the machine commands shown.
Increasing the force on one side however, leads to wavy edge or localized buckling during the rolling process by increasing the strain in the region of additional force. With an average tension applied to the strip by the winder/coiler, the buckle is many times impossible to see. Upon removal of externally applied tension, the excess strain manifests itself in the form of centre-buckles, edge waves, or more complex flatness defects. An especially problematic condition in which buckling occurs, and the motivation for this work, is due to the mis-correction of flatness control systems when rolling and winding strip with wedge type thickness profile. Wedge of even two or three percent of nominal strip thickness can present a major challenge in the stability and quality control of cold rolling operations.
When winding a strip containing wedge, the thickness variation causes stress gradients across the strip width that are transient, and therefore based on mandrel wrap number. This results from the additional distance travelled by thicker regions of the strip during winding. As the number of mandrel wraps grows, the difference between the maximum and minimum diameters of the wound strip (corresponding to thickest and thinnest edges) becomes increasingly significant. If the relative wedge is large enough, the sensor will begin to detect and correct for a force distribution dominated by the winding stress gradient rather than the intended rolling stress gradient. Hence, by not being able to distinguish winding stresses from deformation stresses caused by the mill, the flatness control system is unable to make the correct adjustments to the mill’s flatness control devices.

When the strip is later unwound (free of applied tension) for stamping or slitting operations, the mis-corrected flatness defects are often clearly visible. To date, no calculation method has yet been published that is suitable for incorporation into real-time flatness control systems for the purpose of predicting and correcting for transient winding stresses. The presented work addresses this need through the development of a modified fourth-order Airy function. The analytical Airy function enables rapid calculation of the transient in-plane stress field according to wrap number (which correlates to strip geometry and amount of deflection). Using this approach, the winding stresses can be computed in real time, and filtered from the flatness sensor outputs prior to correction by flatness control systems.
1.4 Prior Winding Models

It is important to note that models for the winding as well as the region of the strip between the mill and mandrel are to be created. This section discusses the past research involved with creating winding models (effects that occur at the mandrel). While several winding stress models have been proposed, they do not rigorously address asymmetric strip thickness profiles or their impact on flatness control systems. The first winding model was developed by Sims and Place in 1953 [3]. Using a model based on the assumptions made by Inglis[4] for wire winding of a gun barrel, they related the tension with tangential and radial stresses using logarithmic approximations. Wilkening later found that the Sims and Place model was insufficient for windings at wrap numbers greater than fifty-five [5]. He determined that the Sims and Place model drastically over predicted stress by more than two hundred percent. By using an empirical correction which imposed a false inner mandrel radius, Wilkening was able to reproduce the experimental results obtained by Sims and Place.

An analytical model, which assumed constant material properties when describing the roll, was developed by Altmann in 1968 [6]. Altmann’s model also only considered radial effects of the winding. While the circumferential stress was allowed to vary, the strip profile was considered uniform. Altmann’s solution was popular because it simplified calculations to integrals and helped explain some of the radial phenomena that occurred during winding. Sptiz studied caliper variation in wound paper rolls using a simplified integral formulation in 1969 [7]. In 1977, Wadsley and Edwards also studied radial stress behavior in the winding, but no strip thickness variation was taken into
account [8]. They improved the “shrink-ring” model by increasing the radial compressibility or contact behavior between wraps. These modifications gave results closer to those of Wilkening’s experimental results. Yagoda subsequently employed a series solution to model the winding near the core [9]. Although more realistic, the method was complex and difficult to apply in real-time flatness control systems. In 1987 and 1992, Hakiel presented a model using the finite difference method to predict the radial and circumferential stresses [10-12]. Although it added the mandrel radial contraction aspect, Hakiel’s model tended to over predict interlayer pressure and did not account for tension losses in the centre of the web. His model also required an initial guess for the radial modulus without knowing the interlayer pressures within the roll itself.

Kedl created a model based upon wrapping a series of thick walled cylinders on to one another [13]. By changing the radial modulus as a function of pressure, Kedl was able to obtain correlating results with experimental data. In 1995, Benson also created a nonlinear model and by using experimental material properties, arrived at more realistic results compared to Hakiel’s model [14]. Recent models include the use of two dimensional axisymmetric finite element analysis such as the work done by Lee, Lin, and Wickert in 2002 and 2003 [15-16]. An inverse solution for non-linear materials was created by de Hoog, et al. and was compared to Benson’s forward solution [17]. The comparison showed that nonlinear properties were not necessary when winding “harder” materials such as steel and aluminium. Liu used displacement based formulation in order to determine strains and stress throughout a winding [18]. This model linearizes the
process by setting up a series of equations that populate a sparse matrix that can be solved for efficiently.

1.5 Prior Planar Models

The second model that is to be constructed consists of the relatively planar region of the strip between the mill and the mandrel. The most common method of determining the stress distribution in a plate that has stress boundary conditions applied to it is formulating an Airy stress function while deriving sets of Fourier series [19-22]. This method allows for any number of boundary conditions to be applied to each of the four sides of a thin plate. In order to obtain results however, the user must know the theory behind solving systems of partial differential equations. Because the boundary conditions can be extremely cumbersome and time consuming to satisfy, a new and more intuitive method is studied. The method of using a series solution is very well known and frequently published, however it is mostly used for buckling analysis and the filtering of winding effects is not discussed. Figure 9 shows an example of in-plane stress distributions using the Fourier series solution [22].

Figure 9: In-plane stress distribution (a)σx (b)σy (c) τxy, Kim (2009)
Large in-plane shear stress gradients are often attributed to winding of strip containing wedge. The adverse effects of in-plane shear stress were studied and shown to play a significant role in causing “cross-buckle” flatness defects in cold rolling. Therefore, in addition to determining normal stresses in the strip, this paper also focuses on predicting in-plane shear stress values. In 1990, Ishikawa, Yukawa, and Hanai separated shearing effects by filtering cross-buckle and macroscopic defects from the strip [23]. The model showed the detrimental effects of stress fields having high amounts of varying shear stress across the width of the strip and how they contribute to flatness problems after rolling. Komori (1993) developed an analytical method using the residual stress distribution in the strip to determine stress fields [24].

The method discussed populates a 4th order Airy stress function using displacement based boundary conditions and the geometry of the strip to arrive at a function that predicts in-plane strain. From these strains, stresses can be determined by multiplying by the material properties, E and G. This function determines strain for the reason that displacement boundary conditions are being used, not stress boundary conditions (used when solving distributions shown in Figure 9).

1.6 Rolling/Winding Model Framework

As stated earlier, this work has multiple facets that pertain to the cold rolling process. Models predicting winding and planar behaviour are used to filter any adverse effects from control system measurement. These two portions are then combined with a rollstack deflection model created by Malik and Grandhi in 2008 [25]. This model
predicts rolling mill force and deflections as well as exit profile of the strip. Once all three of these tools are combined and integrated with one another, a complete simulation of the rolling/winding process can be performed. This gives the mill operator an opportunity to analyze the process before the actual operation takes place, decreasing the opportunity of strip defects, mill over-correction, or any other issues that may arise during manufacturing. Figure 10 depicts the framework of the finished simulation tool showing the three main models that comprise the computing environment.

Figure 10: Simulation of Rolling/Winding Operation Environment
2.0 WINDING MODEL DEVELOPMENT

In order to arrive at improved estimates of stress distributions throughout the strip it is important to formulate a model that can approximate the characteristics/aspects of an actual winding. Although windings are spiral, they are modeled as a series of concentric hoops. With each wrap having a large amount of tension applied causing high magnitudes of stress, the interaction between layers of the winding is very difficult to approximate and can be computationally prohibitive. Surface contact conditions, radial contraction, modulus of the winding itself, interlayer movement/slip, etc. add to the complexity of the model.

2.1 Winding Model Assumptions

With so many aspects involved with a wound coil, a list of major assumptions is made to simplify the system.

Major Assumptions

- *The tangential stress at the coil is equal to the planar stress in the rolling direction at the coil-planar interface* ($\sigma_r = \sigma_\theta$)
- *Each layer is modeled as pre-stressed ring and concentrically wound unto the previous layer*
• The coil is discretized across its width into \( j = 1 \) to \( n \) segments as to account for radial and tangential stress variations during the winding process
• The total amount of tangential stress is dependent on the external tension applied by the winder
• Continuous interlayer contact is not required
• The modulus of the coil is said to equal that of the material that is being wound and is to remain constant
• Although radial contraction can occur, no lateral slip between layers is allowed (no side to side movement)

2.2 Winding-Planar Model Interaction

Although the winding and planar models are handled somewhat separately, they are both used in each model in order to insure continuity as well as adding a more realistic aspect to the final model. An example of this is the calculation of initial tangential stress variations for each wrap. Even though the focus is on the winding, the length of the strip from the mill to the mandrel is needed in order to calculate a closer approximate value of tangential stress. Equations 1 – 5 show the formulation of tangential stress. Figure 11 illustrates the combination of strip length and circumference of the outermost wrap.

![Diagram of winding-planar model interaction](image)

Figure 11: Winding-Planar model interaction/dependency
For each tangential stress calculation the total length (L) of the strip is used, not the circumferential length (C) alone. Equations 1 and 2 define the total length of each strip used to analyze every wrap added to the winding. Note that $\bar{r}$ is a one-time approximation per wrap and is the average radius before and contraction has occurred. With less than 1% change in radial values from contraction, this approximation is sufficient to make.

\[
C_{(i+1)} = 2\pi \bar{r}_{(i+1)} \\
L_{(i+1)} = L_{MM} + C_{(i+1)}
\]

2.3 Modelling Radial Contraction

The basis for radial contract employs Lame’s equations for thick and thin-walled pressure vessels. The geometry of the winding lends itself to thin-walled theory with the thickness of the strip being very small compared to the radius of the mandrel. However, as the coil accrues layers, the “wall thickness” of the coil requires the use of thick-walled theory as well. Using both thin and thick-walled theory is an attempt to capture the transient change in size of the coil during the winding operation. A general depiction of the tangential and planar stress is shown in Figure 12. Note that $\sigma_x$ denotes a Cartesian coordinate direction as it pertains to the planar region of the strip between the mill and mandrel.
Figure 12: Relationship between planar stress and tangential stress

In order to acquire an appropriate, initial tangential profile across the width of the coil a simple strain calculation is used to by finding the difference between actual radial values and the average radius for that wrap number. Equations 3 and 4 define the initial values of tangential stress across the width of the winding \((j)\) for a single layer \((i+1)\). Note that Equation 3 shows the dependency between the winding and strip when determining \(\varepsilon_l\) values as discussed previously. Equation 5 defines the limit of tangential stress as equal to that of the nominal stress applied by the winder. Also, the strip itself cannot handle any compressive stresses in the rolling direction during the winding process. That is not to say that the strip does not have compressive stress induced by the mill, it simply implies that the planar region between the mill and mandrel cannot support a compressive load acting in the plane of the strip in the rolling direction. This will be discussed further in the Planar Modelling portion of this paper.

\[
\delta_l(i+1,j) = \left[r_{(i+1,j)} - r_{avg(i+1)}\right] * 2\pi
\]  
\[
\varepsilon_l(i+1,j) = \frac{\delta_l(i+1,j)}{L_{(i+1)}}
\]  
\[
\sigma_l(i+1,j) = E * \varepsilon_l(i+1,j) = \sigma_{\theta(i+1,j)}
\]
Radial contraction is found using a combination of thin and thick-walled theory. Because the thickness of the strip is much smaller than the radius of the mandrel or coil, the thin-walled assumption is used to determine the amount of pressure exerted on the coil. The external pressure is found using Equation 7. Assuming there is no internal pressure on the inside surface of the coil, the radial contraction is found using Equation 8. Global contraction encompasses the entire winding from the radius of the initial wrap to the radius of the outermost completed wrap \((i)\). This is to model the coil as a single thick-walled cylinder under an applied external pressure at various stages throughout the winding process. As the coil grows in diameter, the magnitude of radial contraction decreases unless the system is altered by unwanted effects such as strip profile variations or applied tension fluctuations. Figure 13 shows a coil with an externally applied pressure.

\[
\sum_{j=1}^{n} \sigma_{\theta(i+1,j)} * a_j = T_{\text{nom}}
\]  

**Figure 13: External pressure acting on coil**
The equations used to define the internal/external pressures and deflections are shown below and are used to determine radial contraction of the coil [26]:

\[ p_{i(i+1)} = \frac{\bar{\tau}_\theta(i+1)}{r_{(i+1)}} = p_{o(i)} \]  \hspace{1cm} (7)

\[ u_{(i)} = \frac{-\left(r_{(i)}^2 p_{o(i)}\right)}{E \left(r_{(i)}^2 - r_{(1)}^2\right)} \times \left[ (1 - \nu) + (1 + \nu) \frac{r_{(1)}^2}{r_{(i)}^2} \right] \]  \hspace{1cm} (8)

It is important to note that Equations 7 and 8 are for the two dimensional case. When computing the radial contraction across the width of the coil these three equations are used for each segment \((j)\) that comprises the total width of the winding. Deriving the radial contraction using simple algebraic equations contributes to the speed and efficiency of the model. It is easy to see that without variation in tangential stress no deviations in radial contraction will be obtained.

2.4 Modelling Interlayer Contact

When accruing wraps on to the mandrel, 100\% interlayer contact is not guaranteed. Gaps are able to form due to lack of tension, non-ideal strip profile, or surface defects such as a large buckle or tear. An example of gapping due to a non-ideal profile is the case in which the strip profile takes on a sinusoidal shape. Although consistent, the severity of the profile limits the total amount of interlayer contact that can be achieved given a nominal amount of applied tension as the number of wraps increases. Figure 14 illustrates this profile and the gaps that can occur due to its non-linearity.
Figure 14: Gapping caused by non-ideal profile (X's denote gapped region), exaggerated

The winding model deals with contact issues by initially assuming the outermost layer makes full contact with the previous wrap. Keep in mind that the system is governed by the amount of externally applied tension provided by the winder (Equation 6). Using this in conjunction with Equations 3 and 4, it is possible to iterate through values of $r_{avg}$ until a single average radius is found that satisfies Equation 6 for that wrap number ($i+1$). Once an average radius is established, a final calculation of tangential stress can be found. This means that some regions of the strip may have no tension/stress acting on them and may not make contact with the previous layer. For these regions, the radius is set to equal that of the smallest radius that has tension applied to it. This is not the same as the average radius, however it does raise the initial radial guess in that region to a more appropriate value as to model a gap in the winding. Figure 15 illustrates a sinusoidal profile with the outermost layer modeled with full contact between layers as well as that same profile with contraction and gapping added to the model (after 100 wraps).
The winding model used for this work assumes no interlayer slip, which is not completely accurate with actual coils. Friction plays a role in determining radial contact conditions and positions. Variations in circumferential stress will cause a longitudinal stress that is also not taken into consideration with this model. Discretizing the width of the coil reduces its accuracy compared to an actual coil, however, this method gives an approximation in very little time and slightly improves the displacement boundary condition used in the planar model. Improvement in the contact conditions and contraction approximations can be made in order to arrive at even more realistic radial positions and stresses in the coil.

Figure 15: Radius of coil after model refinement (Contraction and Contact Conditions)
3.0 PLANAR MODEL DEVELOPMENT

The region between the mill and the mandrel is to be modeled as a two-dimensional plane. It is simple to discretize this region in the same manner the coil was segmented. However, segmenting the strip into long slender pieces is not completely realistic. This results in the loss of any continuum information such as the full two-dimensional stress state of an element within the strip. Although computationally efficient, this method gives an average amount of planar stress in the rolling direction and should be regarded as a rough approximation. However, because it will give ballpark results it is used in comparison to FEA as well as the new method of deriving an Airy stress function discussed later in this section.

3.1 Coordinate System for Planar Model

The planar model uses Cartesian coordinates and takes the centre of the mill bite to be the origin. The mill bite being the location in which work rolls are in contact with the strip. A diagram showing the 3 axes is shown in Figure 15. Note that the $x$-axis coincides with the direction of travel and the middle of the strip. Figure 16 is a top view showing the length of the strip ($L_{MM}$) as well as width ($W$).
3.2 Strip Modeled Using Independent Segments

Major Assumptions

- Strip segments are fixed at $x = 0$
- Adjacent segments are independent of one another
- Amount of stress is limited by applied tension
- Segments can only approximate average normal stress in the rolling direction ($\sigma_x$)
The fastest and easiest way to arrive at an approximation of stress distribution is to continue the coil model by imposing the displacement occurred at the mandrel to the portion of the strip between the mill and mandrel. Modifying Equations 3 and 4, the variation of average stress in the rolling direction across the strip can be found using the equations shown below:

\[
\delta_{x(i+1,j)} = \left[ r_{(i+1,j)} - r_{avg(i+1)} \right] \times 2\pi \times \frac{L_{MM}}{L_{(i+1)}} \tag{9}
\]

\[
\varepsilon_{x(i+1,j)} = \frac{\delta_{x(i+1,j)}}{L_{MM}} \tag{10}
\]

\[
\sigma_{x(i+1,j)} = E \times \varepsilon_{x(i+1,j)} = \sigma_{\theta(i+1,j)} \tag{11}
\]

Equation 9 calculates the correct amount of deflection applied to the strip at \( x = L_{MM} \) in order to make Equation 11 true. Because Equation 3 calculates the total amount of deflection for a strip of length \( L_{(i+1)} = L_{MM} + C_{(i+1)} \), the amount of deflection found must be scaled when applying it to the strip at a different location in order for the average rolling direction (\( \sigma_x \)) and tangential (\( \sigma_\theta \)) stresses to equal each other at the location in which the deflection is being applied, in this case \( x = L_{MM} \). The additional fraction in Equation 9 is simply a scaling factor for the deflection found earlier and the correct proportion of deflection is applied to a length of \( L_{MM} \), not \( L_{(i+1)} \). These approximations, of course, are found using the equations that determine average stress due to a tip deflection. By using independent segments, there is no way to approximate the remaining in-plane stress components, \( \sigma_y \) and \( \tau_{xy} \). Because of its very intuitive results however, this method is employed to compare against more non-intuitive methods such
as FEA and classical elastic theory (Airy function). Figure 18 shows the segmented strip as well as the planar deflection/elongation.

![Segmented strip and planar elongation across width](image)

Figure 18: Segmented strip and planar elongation across width

### 3.3 Strip Modeled Using Finite Elements

**Major Assumptions**

- Strip nodes are fixed at $x = 0$, no deflection in $x$ or $y$
- Strip is modeled as a continuum
- There is no deflection in the $y$-direction at $x = L_{\text{MM}}$
- Segments can only approximate average normal stress in the rolling direction ($\sigma_x$)
- The entire region of the strip between the mill and mandrel is said to lie in a single plane
- The deflection applied to the strip at $x = L_{\text{MM}}$ is proportional to the deflection of the entire strip (entire strip having length $L_{(i+1)} = L_{\text{MM}} + C_{(i+1)}$)

#### 3.3.1 Three Dimensional FEM

A study of the planar region is done using two and three dimensional FEM’s. First, a three dimensional analysis of a strip is done using 4 – node tetrahedral elements.
The entire geometry is modeled: length ($L_{\text{MM}}$), width ($W$), and profile. Figure 19 illustrates an example of the three dimensional geometry used for the analysis. Note that the thickest and thinnest portions of the strip are ($T$) and ($t$) respectively.

![3D geometry of strip used for FEA](image)

**Figure 19: 3D geometry of strip used for FEA**

Once the geometry was defined, the boundary conditions were assigned via the *assumptions* and a deflection boundary condition. Figure 19 depicts the strip with applied displacement boundary conditions. The strip is fixed along the edge $x = 0$ while the displacement is applied along the edge $x = L_{\text{MM}}$. Figure 20 shows a linearly varying displacement boundary condition, however parabolic boundary conditions will also be studied in this work.
3.3.2 Two Dimensional FEM

In order to perhaps save computational time and simplify the model, a two dimensional FEM is built and compared to the results from the three dimensional case. For the comparison case, the geometry and boundary conditions used are identical to those used for the three dimensional model. The only difference being an average thickness \( t \) is used instead of the actual varying thickness profile. The boundary conditions are prescribed per Figure 20. This model employs 4 – node plane stress elements. These elements only pertain to in-plane stresses while any out-of-plane stresses are said to be 0.

The boundary conditions are applied to the nodes along each edge. The displacements along \( x = L_{MM} \) are discretely applied at known nodal locations along the edge. After applying the boundary conditions and completing a simulation, a deformed shape of the strip is obtained. Figure 21 shows an example of the deformed shape after a
simulation has been completed. Note the displacement boundary condition along the edge \( x = L_{\text{MM}} \) is applied across the entire width of the strip.

![Deformed shape of strip after simulation](image)

**Figure 21:** Deformed shape of strip after simulation (dotted lines are un-deformed shape)

### 3.3.3 Two and Three Dimensional FEM Comparison

After completing simulations using two and three dimensional FEM’s with the same geometry and boundary conditions, a comparison between each models’ stress distributions is made. Table 1 defines the geometry and boundary conditions used for both simulations. Table 2 shows the comparison of results recorded at 5 random locations throughout each strip for \( \sigma_x \) values. Similar results were obtained when comparing \( \sigma_y \) and \( \tau_{xy} \) values.

<table>
<thead>
<tr>
<th>Table 1: Geometry and boundary conditions used in FE simulations (inches)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Length (L_{\text{MM}})</strong></td>
</tr>
<tr>
<td><strong>Width (W)</strong></td>
</tr>
<tr>
<td><strong>Max Thk. (T)</strong></td>
</tr>
<tr>
<td><strong>Min Thk. (t)</strong></td>
</tr>
<tr>
<td><strong>Avg Thk. (\bar{t})</strong></td>
</tr>
<tr>
<td><strong>Max Displ. (\delta^N)</strong></td>
</tr>
</tbody>
</table>
Table 2: Comparison between 3D and 2D values for $\sigma_x$

<table>
<thead>
<tr>
<th>Location $(x,y)$</th>
<th>3D (Ksi)</th>
<th>2D (Ksi)</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(10,-5)$</td>
<td>32.73</td>
<td>33.56</td>
<td>2.53%</td>
</tr>
<tr>
<td>$(20,-5)$</td>
<td>101.84</td>
<td>99.12</td>
<td>2.66%</td>
</tr>
<tr>
<td>$(15,4)$</td>
<td>61.19</td>
<td>58.86</td>
<td>3.8%</td>
</tr>
<tr>
<td>$(10,0)$</td>
<td>59.38</td>
<td>60.43</td>
<td>1.77%</td>
</tr>
<tr>
<td>$(15,-4)$</td>
<td>66.76</td>
<td>65.23</td>
<td>2.29%</td>
</tr>
</tbody>
</table>

Average Difference 2.61%

After mesh convergence within 1%, there is a difference of approximately 2% between the 3D and 2D FEM’s. This shows a marginal difference between the two models, however the choice was made to run 2D simulations to model the strip. This lead to the derivation of an Airy stress function to define the stress distributions throughout the strip based on geometry and displacement boundary conditions.

3.4 Airy Stress Function

In using planes stress elements, the goal is to define an Airy function that is to be used in defining the entire stress state throughout the strip. The relationship between the in-plane stresses and the Airy stress function ($\Phi$) are shown in Equation 12 [27]. Satisfaction of elasticity relations requires that the Airy function satisfies the biharmonic relation in Equation 13. All out-of-plane stresses are zero, as indicated in Equation 14. If the plane stress FEM shows to have similar results when compared to the three dimensional FEM, the use of Equations 12 – 14 can be used to define the stress
distributions in the strip. That is, if an Airy function can be obtained/derived, the stress distributions can be found using Equation 12.

\[
\frac{\partial^2 \Phi}{\partial y^2} = \sigma_x \quad \frac{\partial^2 \Phi}{\partial x^2} = \sigma_y \quad -\frac{\partial^2 \Phi}{\partial y \partial x} = \tau_{xy}
\] (12)

\[
\frac{\partial^4 \Phi}{\partial x^4} + 2 \frac{\partial^4 \Phi}{\partial x^2 \partial y^2} + \frac{\partial^4 \Phi}{\partial y^4} = \nabla^4 \Phi = 0
\] (13)

\[
\sigma_z = \tau_{xz} = \tau_{yz} = 0
\] (14)
4.0 DERIVING AIRY FUNCTION

The overall goal discussed in this section is to derive an Airy function based on geometry and displacement boundary conditions of the strip. Because Airy functions are most commonly polynomial expressions depending on $x$ and $y$, they are computationally efficient and a strong candidate to be used during real time applications. By completing many 2D simulations using various strip geometries and boundary conditions, trends are obtained and approximating equations are used to arrive at an Airy function. This function however, defines in-plane strains because the derivation involves displacement boundary conditions, not stress boundary conditions. Airy stress functions are material independent and most commonly have stress or force boundary conditions applied to them. The following Airy function defines strain because when displacing the strip, the stress values are dependent on the material properties of the strip. Displacing a strip of aluminum will cause lower stress values if the strip made were of steel and displaced the same amount.

4.1 Results from Two Dimensional FEA Simulations

A series of FEA simulations was completed in order to obtain information on how the stress distributions behaved under differing boundary conditions. The boundary
conditions were applied in the same manner that was discussed in section 3.3. The stress distributions were recorded at known locations and analyzed using Matlab [25]. An example of the \( \sigma_x \) distribution of a strip is shown in Figure 22. The distribution shown is from the model used for the comparison in section 3.3.3. The recorded \( \sigma_x \) values at known locations are illustrated in Figure 23.

![Contour plot of \( \sigma_x \) distribution](image)

**Figure 22: Contour plot of \( \sigma_x \) distribution**

![Recorded stress values at known locations throughout strip](image)

**Figure 23: Recorded stress values at known locations throughout strip**

As stated previously, a number of simulations were completed and the stress distributions were recorded and analyzed using Matlab. From these analyses, it was possible to obtain approximations for the in-plane strain distributions. Note that all dimensional values were normalized before the analysis was performed. Meaning, the system was reduced to values of strain and non-dimensional positions. This was done by dividing the width by itself and the length by a characteristic length \( (L_C) \) chosen to be 25 inches.
4.1.1 Stress/Strain $x$

The first set of analyses deals with determining how the strain in the rolling direction ($\varepsilon_x$) behaved throughout the strip. Multiple lengths ($L_{MM}$) and displacement values ($\delta$) were used. Values for $\varepsilon_x$ are obtained by dividing $\sigma_x$ by $E$. An example of the $\varepsilon_x$ distribution is shown in Figure 24 as a surface plot. Figure 24 illustrates a non-linear distribution throughout the strip while creating somewhat of a saddle shape. Note that data within 1 unit width of the strip from each end is not included in the analysis to avoid boundary condition effects. The response shown is in regards to a strip with the attributes:

**Length:** 116.04 inches

**Width:** 10 inches

**Max Deflection ($\delta$):** 1.88 inches (varies linearly along edge $x=116.04$)

**Thickness ($t$):** .06325 inches

![Figure 24: Surface plot of $\varepsilon_x$ distribution](image)
In order to further analyze the distribution shown in Figure 24, the traces of the surface plot are viewed with respect to the now normalized \( x \) and \( y \) axes. These traces depict sets of linear behavior across the width as well as down the length of the strip. Figure 25 shows the linear behavior with respect to each axis for the distribution illustrated in Figure 24.

**Figure 25: Linear traces of \( \varepsilon_x \) distribution shown in Figure 24**

The next step is to define the equation of the \( \varepsilon_x \) distribution. This is done by curve fitting each of the lines in the top plot of Figure 25. Each of the six lines takes the form of a linear approximation. Equation 15 defines the form of each line from the top plot of Figure 25. Note: \( k = 1 \) to 6 for this case.

\[
\varepsilon_x = m_k x + b_k
\] (15)
When determining the coefficients \( m \) and \( b \), it is noticed that they too are linear in nature and are dependent on \( y \). They take the form of Equation 15, giving Equations 16 and 17. Note: \( k2 \) and \( k3 = 1 \) to 6.

\[
m_k = m_2 y + b_2 \tag{16}
\]
\[
b_k = m_3 y + b_3 \tag{17}
\]

Substituting Equations 16 and 17 into 15 gives Equation 18a. This equation defines the entire \( \varepsilon_x \) distribution for the strip given the geometry and boundary conditions stated previously. The coefficients (\( m \)'s and \( b \)'s) are solved using the curve fitting toolbox within Matlab. From these values it is determined that the \( b_2 \) coefficient is relatively small compared to the remaining coefficients. Removing \( b_2 \) from Equation 18a results in a change of less than 1%. Equation 18a then simplifies to Equation 18b.

\[
\varepsilon_x = b_2 x + m_2 xy + m_3 y + b_3 \tag{18a}
\]
\[
\varepsilon_x = m_2 xy + m_3 y + b_3 \tag{18b}
\]

*Where: \( m_2 = -.01985 \), \( m_3 = .02246 \), and \( b_3 = .00807 \)*

It is important to note that this method/algorithm is completed for each individual simulation. Equation 18 gives a single piece of the “Airy Puzzle” in which we are trying to complete. The two remaining stress/strain components (\( \varepsilon_y \) and \( \gamma_{xy} \)) are required to complete the overall Airy function that we strive to obtain. For the sake of clarity, let’s assume we only do a single simulation and derive the Airy function for that specific case. In reality, many simulations are completed to arrive at multiple Airy functions pertaining to different geometries and boundary conditions. These cases will be discussed further.
later in this paper, but for now, we only care about the single $\varepsilon_x$ distribution defined in Equation 18.

### 4.1.2 Stress/Strain $y$

When recording $\sigma_y$ values, it is quickly noticed that all of them have a magnitude close to 0. From this we, assume that $\sigma_y=0$ for the strip under the given boundary conditions. That is not to say that $\varepsilon_y$ is 0. From Ugural and Fenster, the equation for $\varepsilon_y$ in a plane stress condition is [26]:

$$
\varepsilon_y = \frac{1}{E} (\sigma_y - \nu \sigma_x) \tag{19}
$$

$$
\sigma_y = 0
$$

$$
\varepsilon_y = \frac{-\nu \sigma_x}{E} \tag{20}
$$

With $\varepsilon_y$ dependent only on $\varepsilon_x$ via Equation 20, it is left out of the Airy function derivation all together. If one wanted to determine the widthwise strain ($\varepsilon_y$), Equation 20 would be used after calculating the lengthwise strain ($\varepsilon_x$). This simplifies the Airy derivation by reducing the number of terms that need to be defined. Figure 26 illustrates the widthwise strain distribution ($\varepsilon_y$).
4.1.3 Shear Stress/Strain $xy$

Dealing with the same system as in the previous two sections, the shear stress is recorded and then divided by $G$ in order to obtain shear strain values. When plotting the distribution of shear strain, a parabolic behavior is obtained. Figure 27 depicts the shear strain distribution of the strip. Figure 28 illustrates the traces with respect to the $x$ and $y$ axes. It is noticed that the shear strain is constant down the length of the strip and parabolically decreases to 0 across the width of the strip. This is due to the imposed tip load by not allowing any deflection in the $y$ direction to occur. By not allowing any $y$ deflection to occur along $x=116.04$, the strip acts as if a tip load is applied along the edge $x=116.04$. This tip load correlates to a parabolic shear stress distribution across the width [27].
Figure 27: Surface plot of shear strain

Figure 28: Traces of shear strain with respect to $x$ and $y$ axes
Figures 27 and 28 show the constant lengthwise behavior of shear strain as well as the parabolic widthwise behavior. Because the values remain constant down the length of the strip, it is determined that the shear strain is a function of $y$ only. Again, the curve fitting toolbox in Matlab is used to determine the coefficients to the parabolic equation shown below:

$$\gamma_{xy} = Ay^2 + By + C$$

(21)

It is determined by the relative magnitude of $B$ compared to those of $A$ and $C$ that $B$ is not a major component is the shear strain approximation. This makes sense seeing as the $B$ coefficient determines the magnitude of the linear component in the equation.

From the bottom plot of Figure 28, there does not appear to be a linear component in the shear strain distribution. This simplifies Equation 21 to:

$$\gamma_{xy} = Ay^2 + C$$

(22)

Where: $A = -.01017$ and $C = 2.58e-03$

4.2 Airy Function Derivation

Now that the two components ($\varepsilon_x$ and $\gamma_{xy}$) of the Airy function have been determined, a single Airy function can be obtained. Using the following algorithm, an Airy function that defines in-plane strain distributions is determined for the given case.
Starting with Equation 12:

\[
\frac{1}{E} \frac{\partial^2 \Phi}{\partial y^2} = \sigma_x = (m_2 xy + m_3 y + b_3)
\]  
(23)

\[
\Phi = \iint \epsilon_x \, dy \, dx = \iint (m_2 xy + m_3 y + b_3) \, dy \, dx
\]  
(24)

\[
\Phi = \left( \frac{m_2}{6} xy^3 + \frac{m_3}{6} y^3 + \frac{b_3}{2} y^2 \right)
\]  
(25)

Moving to Shear:

\[
-\frac{1}{G} \frac{\partial^2 \Phi}{\partial y \partial x} = \frac{\tau_{xy}}{G} = (Ay^2 + C)
\]  
(26)

\[
\Phi = \iint \gamma_{xy} \, dx \, dy = \iint (Ay^2 + C) \, dx \, dy
\]  
(27)

\[
\Phi = \left( \frac{A}{3} xy^3 + C xy \right)
\]  
(28)

Leaving two potential Airy functions:

\[
\Phi = \left( \frac{m_2}{6} xy^3 + \frac{m_3}{6} y^3 + \frac{b_3}{2} y^2 \right)
\]  
(25)

\[
\Phi = \left( \frac{A}{3} xy^3 + C xy \right)
\]  
(28)

If the following substitution can be made, a single Airy function is obtained

\[
A = \frac{m_2}{2}
\]  
(29)

When performing this approximation it is found that:

\[
A = \frac{m_2}{2} = -\frac{0.01985}{2} = -0.009925
\]  
(30)

\[
\frac{m_2}{2} = -0.009925 \approx -0.01017 = A
\]  
(31)

Making the substitution a single Airy function is obtained:
After substituting numerical values:

\[
\Phi = \left( \frac{m_2}{6} xy^3 + \frac{m_3}{6} y^3 + \frac{b_3}{2} y^2 + Cxy \right) \quad (32)
\]

After substituting numerical values:

\[
\Phi = \left( -0.01985 \frac{xy}{6} + \frac{0.02246}{6} y^3 + \frac{0.00807}{2} y^2 + 2.58 \times 10^{-3} xy \right) \quad (33)
\]

This algorithm is used for multiple different geometries and boundary conditions. The goal now is to determine the Airy function for any sized strip with any continuous linearly varying displacement boundary condition. This is done by first noticing that the fourth coefficient in Equation 33 correlates to the \( C \) coefficient in Equation 22. This coefficient defines the maximum value of shear strain for the entire strip. The maximum value occurring at \( y=0 \), down the length of the strip. Because this value is easily monitored through FEA simulations, it will be used in determining the remaining coefficients in Equation 32 via a second algorithm discussed later. For now, the maximum amount of shear strain is recorded using multiple geometries and displacement boundary conditions. After many simulations are completed, the maximum amount of shear strain is plotted against the aspect ratio (AR) as well as another dimensionless value (\( \delta^* \)). Equations 34 and 35 define these two values. Figure 29 shows the maximum amount of shear strain for various aspect ratios.

\[
AR = \frac{L_{MM}}{W} \quad (34)
\]

\[
\delta^* = \frac{\delta}{2W} \quad (35)
\]
The behavior of $C$ is also plotted against $\delta^*$. This is done to see if there is any strong correlation between the two values like the ones shown in Figure 29. Figure 30 shows the response of $C$ when holding AR constant (various constant values) and varying $\delta^*$. From Figures 29 and 30 it is found that $C$ has an inverse relationship with respect to AR and a linear relationship with respect to $\delta^*$. 

Figure 29: Value of $C$ for various aspect ratios (AR)
Knowing that the formula for $C$ most likely involves a fraction $\delta^*$ in the numerator and AR in the denominator, trial and error is used until an approximation for $C$ is sufficient for use in this model. The equation defining the value of $C$ as a function of $\delta^*$ and AR is shown below:

$$C = \frac{4\delta^*}{(AR + .5)^2}$$  \hspace{1cm} (36)

Equation 36 is then tested against many scenarios in order to determine how close the approximating function (Equation 36) comes to predicting the maximum amount of shear in the strip ($C$). For this, 13 random cases were chosen to test the validity of Equation 36. Table 3 shows the results of the test simulations and Figure 31 is a graphical example of max shear strain ($C$) as both AR and $\delta^*$ change in value.
Table 3: Comparison between FEA and Approximation for $C$

<table>
<thead>
<tr>
<th>AR</th>
<th>$\delta^a$</th>
<th>Approx. Max Shear Strain (C)</th>
<th>FEA Max Shear Strain (C)</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.1429</td>
<td>.0044767</td>
<td>0.000830375</td>
<td>.0008666</td>
<td>4.2%</td>
</tr>
<tr>
<td>5.71377</td>
<td>.023325</td>
<td>0.002416414</td>
<td>.00256</td>
<td>5.6%</td>
</tr>
<tr>
<td>7.677</td>
<td>.046885</td>
<td>0.002804826</td>
<td>.00296</td>
<td>5.2%</td>
</tr>
<tr>
<td>11.604</td>
<td>.0940</td>
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<tr>
<td></td>
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<td>Avg. Error</td>
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The average error between FEA and Equation 36 is approximately 3.6% for the test cases completed. At lower AR values, the responses seem to be not as consistent compared to the higher AR values. This may be due to boundary conditions effects. With this information, the second algorithm can be employed to determine the remaining coefficients in the Airy functions.
Now that $C$ has been defined (Equation 36), more of the coefficients in the Airy function can be determined. Starting with the shear-strain equation (22), it is known that the shear strain decreases to 0 along the edges of the strip. Now that the maximum shear strain ($C$) is defined, the coefficient of ($y^2$) can be solved for. The following steps show this in algebraic form:

$$\gamma_{xy}(x, y = 0) = C$$

(37)

Knowing the shear strain goes to 0 along both edges:

$$\gamma_{xy}(x, y = \pm 0.5) = A(\pm 0.5)^2 + C = 0$$

(38)

Meaning $A$ is equal to:

$$A = -4C = \frac{m_2}{2}$$

(39)

Making 2 of the 4 coefficients now determined.
Now that the shear strain is fully defined, the two remaining coefficients are to be solved for. By observation, the $b_3$ coefficient is found to equal the average strain throughout the strip. Equation 40 shows this relation.

\[ b_3 = \frac{\delta_{avg}}{L_{MM}} \]  \hspace{1cm} (40)

The final coefficient of the Airy function (Equation 32) to be determined is $m_3$. This term is the least obvious or intuitive, and in order to approximate $m_3$ a new variable is defined. Inflection ratio ($\overline{IR}$) is the normalized position of the inflection location (shown in Figure 25). First, IR (un-normalized) is plotted against various strip lengths ($L_{MM}$). Figure 32 shows the values for $\overline{IR}$ a number of strip lengths. From Figure 32 below, IR is approximated to be .32.

![IR vs Strip Length](image)

**Figure 32: IR for various strip lengths (LMM)**
Equation 41 is used to approximate the $\overline{IR}$ value:

$$\overline{IR} = \frac{.32 \cdot L_{MM} - 10}{25}$$  \hspace{1cm} (41)

With $IR$ now defined it is used in conjunction with the fact that there is no change in $\varepsilon_x$ across the width of the strip ($y$) at the $x$ location $x=\overline{IR}$. The following algorithm was used to solve for $m_3$:

$$\frac{\partial \varepsilon_x}{\partial y} (x = \overline{IR}) = m_2 \cdot (\overline{IR}) + m_3 = 0$$  \hspace{1cm} (42)

$$m_3 = -m_2 \cdot (\overline{IR})$$  \hspace{1cm} (43)

Because this study is done using a strip width ($W_C$) of 10 inches, a scale factor for $m_3$ must be used. The scale factor is simply the ratio of $W_C$ to $W$, where $W$ is the strip width of the current simulation. Equation 44 illustrates the use of this ratio:

$$m_3 = -m_2 \cdot (\overline{IR}) \cdot \frac{W_C}{W}$$  \hspace{1cm} (44)

*Now that all coefficients have been solved for, the Airy function is:*

$$\phi = \frac{-8 \cdot 4 \delta^*}{6(AR + .5)^2} xy^3 + \frac{-m_2 \cdot (\overline{IR}) \cdot W_C}{6W} \frac{\delta_{avg}}{L_{MM}} y^3 + \frac{\delta_{avg}}{2(AR + .5)^2} y^2$$  \hspace{1cm} (45)

Equation 45 is based upon strip dimensions and the applied boundary condition. This allows for rapid computation as well as a wide range of system configurations. Note that the boundary condition applied along the edge $x=L_{MM}$ is of low order (1st or 2nd).

This method has not been validated to work on higher ordered or trigonometric boundary conditions.
4.3 Airy Function Validation

Now that an Airy function has been derived, multiple cases have been chosen to test its validity and accuracy. Both linear and parabolic displacement boundary conditions are used to test Equation 45. It is first compared to FEA results using a linear boundary condition, such as the one previously discussed in this paper. It is then compared against FEA simulations in which the displacement is applied in a nonlinear manner (2nd order). This is more realistic as a strip never truly has a linearly varying profile.

4.3.1 Linear Boundary Condition

This section pertains to the error analysis between Equation 45 and FEA simulations. After applying a nominal amount of linearly varying displacement along \( x = L_{\text{MM}} \), the stress distributions throughout the strip are compared to the distributions approximated by Equation 45. An example of the process is shown, but a table of numerous test case comparisons will be given at the end of this section.

To start, the strip geometry is chosen as well as the amount of displacement that is to be applied.

**Length:** 135.0 inches \((L_{\text{MM}})\)

**Width:** 13 inches \((W)\)

**Max Deflection \((\delta)\):** 1.75 inches \((\text{varies linearly along edge } x=135.0)\)

The FEM is built using the information shown above with plane stress elements as done previously. The stress distributions are recorded and used later to compare with
approximated values. Figure 33 shows the geometry along with boundary conditions.

Figure 34 shows the normalized strip that is used for analysis.

Figure 33: Schematic showing geometry and boundary conditions

Figure 34: Normalized geometry of strip used for analysis
The system must be normalized before employing Equation 45. The following steps are done to non-dimensionalize the system and determine the coefficients:

\[
AR = \frac{L_{MM}}{W} = \frac{135}{13} = 10.385
\]

\[
\delta^* = \frac{\delta}{2W} = \frac{1.75}{26} = 0.06731
\]

\[
C = \frac{4\delta^*}{(AR + 0.5)^2} = \frac{4 \times 0.06731}{(10.385 + 0.5)^2} = 0.002272
\]

\[
m_2 = -8C = -0.018179
\]

\[
b_3 = \frac{\delta_{avg}}{L_{MM}} = \frac{0.875}{135} = 0.006481
\]

\[
L_{MM} = \frac{L_{MM} - 20}{25} = \frac{135 - 20}{25} = 4.6
\]

\[
\bar{IR} = \frac{0.32 \times 135 - 10}{25} = 1.328
\]

\[
m_3 = -m_2 \times \frac{W_c}{W} = -0.018179 \times 1.328 \times \frac{10}{13} = 0.0185
\]

\[
\Phi = \frac{-0.018179}{6} x^3 + \frac{0.0185}{6} y^3 + \frac{0.006481}{2} y^2 + 0.002272 xy
\]

(46)

The strain distributions are:

\[
\varepsilon_x = \frac{1}{E} \frac{\partial^2 \Phi}{\partial y^2} = (-0.018179 xy + 0.0185 y + 0.006481)
\]

(47)

\[
\gamma_{xy} = \frac{1}{G} \frac{\partial^2 \Phi}{\partial y \partial x} = (-0.009089 y^2 + 0.002272)
\]

(48)

The error between FEA results and Equation 45 can now be calculated.
It is found that another scale factor must be used with the $xy$ coefficient in Equation 47. When using a strip width other than 10 inches, this factor must be used. This scale factor is not used in Equation 48. In order to determine $\sigma_x$, Equation 47 goes to:

$$\sigma_x = E \cdot \varepsilon_x = E \cdot \left( -0.018179 \left( \frac{10}{13} \right) xy + 0.0185y + 0.006481 \right)$$ (49)

The error analysis is completed by calculating the percent error between the Airy function and the FEA results. Error is calculated using Equation 50 shown below:

$$error = \frac{\sum \sigma_{x,i} d_i}{\sum \sigma_{x,i}} \cdot 100$$ (50)

Where $d_i$ is the difference between FEA and Airy stresses in the rolling direction.

For the system being analyzed, the average error throughout the analysis portion of the strip ($L_{MM-20''}$) is shown in Table 4. Table 4 shows average amount of error for both $\sigma_x$ and $\tau_{xy}$.

<table>
<thead>
<tr>
<th>Stress Component</th>
<th>Average Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_x$</td>
<td>6.7</td>
</tr>
<tr>
<td>$\tau_{xy}$</td>
<td>4.2</td>
</tr>
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</table>

In order to better visualize the error analysis, surface plots are given. Figure 35 illustrate the behavior of $\sigma_x$ from the FEA simulation for this test case. Note the bottom right corner contains very high magnitudes of stress. The top right and bottom left corners contain negative magnitudes of stress.
Figure 35: Surface plot of $\sigma_x$ values from FEA simulation

A similar surface plot is created when using Equation 49. Figure 36 is the surface plot that is created when using the approximating function. These plots are to give a visual aide only. There are no value bars of legends in order to clearly illustrate overall behavior, not precise values.

Figure 36: Surface plot of $\sigma_x$ using approximating function

A final surface plot is that of the error between the previous two figures. Figure 37 shows that high amount of error lie in the regions in which small magnitudes of stress are located. Meaning, the regions in which the $\sigma_x$ values change from positive to negative tend to obtain more error. This is mainly due to the small magnitudes of stress that divides the difference as in Equation 50.
If the regions of small stress values (100-1000% error) are removed, the average error throughout the strip decreases to approximately 4-5%. This method appears to be sufficient for a linearly varying displacement boundary condition.

### 4.3.2 Parabolic Boundary Condition

The same approximating equation is used for a non-linear displacement boundary condition. Note that now the $\delta_{avg}$ value will be altered due to the behavior of the applied displacement. The overall $\delta$ will remain the same as in the linear case however, the displacement will be applied using a parabolic distribution instead of a linear one. Figure 38 illustrates the difference between linear and parabolic boundary conditions at $x=L_{MM}$. 

![Figure 38: Linear and Parabolic displacement boundary conditions](image)
Similar to the linear case, the system must be normalized in order to arrive at appropriate values for the coefficients. The following steps show the process of deriving the Airy function for the parabolic case.

\[
AR = \frac{L_{MM}}{W} = \frac{135}{13} = 10.385
\]

\[
\delta^* = \frac{\delta}{2W} = \frac{1.75}{26} = .06731
\]

\[
C = \frac{4\delta^*}{(AR + .5)^2} = \frac{4 \cdot .06731}{(10.385 + .5)^2} = .002272
\]

\[
m_2 = -8C = -.018179
\]

\[
b_3 = \frac{\delta_{avg}}{L_{MM}} = \frac{1.1637}{135} = .00862
\]

\[
\bar{L}_{MM} = \frac{L_{MM} - 20}{25} = \frac{135 - 20}{25} = 4.6
\]

\[
\bar{I}R = \frac{.32 \cdot 135 - 10}{25} = 1.328
\]

\[
m_3 = -m_2 \cdot (IR) \cdot \frac{W_C}{W} = -.018179 \cdot 1.328 \cdot \frac{10}{13} = .0185
\]

\[
\Phi = \frac{-0.018179}{6} xy^3 + \frac{0.0185}{6} y^3 + \frac{0.00862}{2} y^2 + .002272xy
\]

\[
The\ stress\ distributions\ are:\
\]

\[
\sigma_x = E \cdot \varepsilon_x = E \left( -.018179 \left( \frac{10}{13} \right) xy + .0185y + .00862 \right)
\]

\[
\tau_{xy} = -G \cdot y_{xy} = G \left( -.009089y^2 + .002272 \right)
\]

Note the ratio of $(10/13)$ in Equation 52. This was found to be needed when using strip widths other than 10 inches, just as in the case for finding $m_3$.

The stress distributions from the parabolic case are very similar to that of the linear case. One can see that the only difference is that of $b_3$. This term changed by
approximately 33% as the distribution of displacement was no longer constantly varying. The average error values between the FEA simulation and Equations 52 and 53 are shown in Table 5. The surface plot of error pertaining to $\sigma_x$ values is shown in Figure 39.

<table>
<thead>
<tr>
<th>Stress Component</th>
<th>Average Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_x$</td>
<td>5.9</td>
</tr>
<tr>
<td>$\tau_{xy}$</td>
<td>3.8</td>
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</tbody>
</table>

Figure 39: Surface plot of percent error between FEA and approximation

As in the linear case, regions near the boundaries or of small magnitudes tend to obtain higher amounts of error. The interior portion of the strip however remains fairly accurate even near the boundaries. Shear stress is also compared. With such a constant behavior down the length of the strip, only the widthwise distribution is plotted. The shear stress distributions (FEA and Airy) across the width of the strip are shown in Figure 40.
Figure 40: Comparison of $\tau_{xy}$ distributions across strip width

Other simulations and error analyses can be viewed in Appendix A.
5.0 SYSTEM MODELING AND EXPERIMENTAL COMPARISON

This section will discuss combining coil contraction and each of the 2D planar models in order to obtain a complete simulation tool that encompasses the three major aspects of cold rolling. First, the discretized-planar approach is used in conjunction with the coil contraction model, then the Airy approximation method is used with the coil contraction model. Both approaches are compared to industry results to see how well each method comes to estimating the system behavior.

5.1 System Definition and Characteristics

The comparison to be made is very much a comparison showing correlation and trend behavior between simulation and industry data. By gathering data from industry we can compare both planar methods to actual measured results. The mill type, configuration, strip geometries rolled are as follows:

Mill Type: ZR 23 CN26 (20-high mill); Strip Width Range: 330–660 mm, Alloy Types: stainless steels; Strip Thickness Range: 2–0.05 mm; Mandrel Diameter: 610 mm; Distance from Mill to Mandrel: 3085 mm (121 inches); Distance from Mandrel to Deflector Roll: 1860 mm; Distance from Deflector Roll to Shapemeter: 335 mm; Shapemeter: U.S. Patent 6,658,947B1. Corresponding to flatness control system data
logs provided by T. Sendzimir, Inc., winding stress predictions using an Airy function are made for a 614.68 mm (24.2) wide stainless steel strip, having 0.23 mm (.0091) nominal thickness. During rolling, a tension force of 34 kN (7643) is applied by the winder. Assuming that the shapemeter is positioned midway between the mill and mandrel, the strain distribution across the strip width only at this position should be considered when filtering winding effects.

According to T. Sendzimir, Inc., it can be assumed that this strip contains approximately 2.0% wedge, although it should be stated that data on the actual thickness profile for this strip was not available. Shapemeter flatness signals, indicating ‘l-units’ (equivalent to strain in the rolling direction, $\epsilon_x$, times $10^5$) are used in industry when monitoring strip profiles. That being said, the comparison will pertain to only a portion of the winding process as the strip profile does change constantly during operation. A field of approximately 50 mandrel wraps is studied in a coil that contains over 2000 wraps.

After approximately 10 wraps of winding 2% wedge material, the shapemeter measurement depicts a slight amount of wedge behavior. This is shown in Figure 41 and when modeling this with both methods (discretized and Airy), the model predictions are overlaid on top of the measured reading. Figure 42a and 42b illustrate model response with discretized and Airy methods respectively.
Figure 41: Shapemeter after approximately 10 wraps of 2% wedge

Figure 42: Predicted winding contribution using discretized (a) and Airy (b) methods (10 wraps)

The shapemeter measurement is also compared to simulation after 50 wraps of winding approximately 2% wedge. Figures 43a and 43b illustrate each model’s behavior after this time. The predicted winding contributions would be subtracted from the shapemeter measurements before the control system adjusts the mill in order to decrease the probability of mis-correction.
The comparison at 10 and 50 wraps for both models show some correlation. It is too early to say which model actually performs better. Although shear stress predictions were obtained using the Airy method, it is unknown whether or not shear stress plays any role in the shapemeter measurement. For this reason, only $\varepsilon_x$ is considered when filtering the shapemeter readings.

### 5.2 Discussion

The discretized an Airy techniques show correlation with industry results. However, the discretized method does not account for multiple sensor positions because it is only the average strain of the segmented strips. Also, because the strips are independent of one another using this method, the strain distributions can look very discontinuous and not realistic. The discretization of the coil itself seems to work adequately, however, when attempting to apply the same method to the planar region the method breaks down.
The above example is a single comparison to industry measurements. Future tests will further validate the derived method before any implementation is to take place. Also, because the entire 2D stress state is defined using the Airy approach, investigations into the effects of shear stress can be done in order to determine any adverse effects caused by shear stress.
6.0 SUMMARY AND CONCLUSION

Presented was a new and rapid method used to predict the 2D stress state of a planar region subjected to linear or parabolic displacement boundary condition. Curve fitting and function approximations are used to determine the coefficients that populate a 4th order Airy function that determines in-plane strain components of the planar region. From this prediction, the contribution of winding effects is able to be filtered from measuring devices used by rolling mills. This filtering can reduce the occurrence of mis-correction of the mill settings and improve the final quality of the strip. Also, knowing the non-uniform elastic strain distribution during winding allows for the adjustment of winding tension if regions of larger elastic strain exist. The operator can reduce the winding tension in order to avoid plastic elongation of the strip due to build up of elastic strain. Thus, optimization of winding tension can be completed for a given strip with a wedge shaped thickness profile.

The derived planar model is used in conjunction with an axisymmetric model of the coil during the winding process. This model accounts for contraction of the coil as well as non-contact conditions during winding. Using Lame’s equation for thick and thin-walled pressure vessels, radial contraction is approximated. The updated coil model is used to improve the boundary conditions applied to the planar region of the strip. These two models can be combined with the rollstack deflection model created by Malik
and Grandhi [25]. The predicted stress/strain field is used to filter elastic winding effects from shapemeter measurements. This complete model can be used to as a tool to assist in the rolling and winding processes.

The Airy method discussed is able to rapidly predict a stress field when a planar region is subjected to a low order displacement boundary condition. The method does not account for 3\textsuperscript{rd}, 4\textsuperscript{th}, or higher ordered boundary conditions. Further research is able to be done in order to expand this method to account for higher ordered effects and predictions. Also, the method discussed uses only continuous boundary conditions. When predicting non-continuous boundary conditions an approximation or simplification must be made in order to account for the discontinuity of the boundary condition.

The compilation of this work has shown to correlate well with industry data and gives industry leaders more insight into perhaps the driving factors and conditions that are involved with this issue. Knowing more about the problem aides in a better solution and provides a theoretical reasoning to physical occurrences during operation.
7.0 BIBLIOGRAPHY


## 8.0 APPENDIX A

<table>
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